Discrete models and algorithms for the capacitated location problems arising in UMTS network planning*

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ABSTRACT

In UMTS networks base station location cannot only be based on signal predictions, but it must also consider the traffic distribution, the power control mechanism as well as the power limits and the signal quality constraints. In this paper we discuss integer programming models and discrete algorithms aimed at supporting the decisions in the process of planning where to locate the new base stations. We consider the Signal-to-Interference Ratio (SIR) as quality measure and two power control mechanisms which keep the received signal power or the estimated SIR at a given target value. The focus is on the uplink (mobile to base station) direction which turns out to be the most stringent one in the presence of full-duplex balanced connections such as voice calls. To find good approximate solutions of these NP-hard problems which are extensions of standard capacitated location problems, we present randomized greedy and reverse greedy procedures as well as a randomized method combining the types of steps used in the two above procedures. Computational results are reported for small to large size realistic uplink instances generated using classical propagation models.

1. INTRODUCTION

Since radio resources are scarce in cellular networks, radio planning is a central issue for system efficiency [7, 14, 17, 18]. Universal Mobile Telecommunication System (UMTS) [16, 15, 4] is the third generation mobile communication system developed by ETSI, the European Telecommunication Standard Institute, which will allow the use of a new frequency spectrum and is expected to extend the present GSM service to include multimedia. With the advent of UMTS the problem of locating and configuring base stations (BSs) is expected to become of particular importance for the operators. Indeed, to guarantee a reasonable service coverage a very large number of new base stations will have to be deployed in just a few years and this will require considerable investments (e.g., approximately $50,000$ base stations and an estimated total of $15,000$ million EURO for the four new UMTS service providers in Italy).

The planning of second generation cellular systems adopting a TDMA-based access scheme is usually carried out by subdividing the problem into a coverage problem and a frequency allocation problem [11, 7, 14, 20]. In the coverage planning phase, base stations are placed so that the signal strength is high enough in most of the service area. This step only makes use of propagation models, such as for instance Hata's model, to predict the signal levels (see for instance [12]). In the frequency planning phase, a number of channels has to be assigned to each BS [6, 2]. This is achieved by taking into account the traffic load as well as the level of service measured as the Signal-to-Interference Ratio (SIR), i.e., the ratio between the relevant signal and the sum of the other signals.

Since the FDD (Frequency Division Duplex) air interface of UMTS is based on W-CDMA (Wideband Code Division Multiple Access), this two-phase approach is not appropriate for at least two reasons. First, the bandwidth is shared by all active connections and no actual frequency assignment is strictly required. Second, the area actually covered by each BS is heavily affected by the traffic distribution. Indeed, the actual capacity of each cell is not specified by the number of connections, as in FDMA systems. A new connection is accepted based on the SIR values achievable by each other existing connection once the new one is activated. These values are functions of the emitted powers which, due to a *power control* (PC) mechanism, depend on the mobile user positions. Since the power available at each BS is limited, the number of users that can be served is large if the former are close to the BS and small if they are far away (the so called *cell breathing* effect). Thus, in UMTS networks, BS location and configuration (e.g., power emission selection) cannot be only based on signal strength predictions, but must also take into account the traffic distribution, the PC mechanism as well as the power limits and the quality constraints.

To the best of our knowledge, there has been so far little work on the optimization of BS location and config-

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uration for UMTS networks. In [10] some mathematical programming formulations based on minimum dominating set are proposed, considering different objective functions. But these formulations only focus on the downlink direction and they do not address the crucial issue of interference in the W-CDMA setting. Indeed, Fixed Channel Assignment (FCA) is considered which is typical of the second generation (GSM) cellular networks. In [5] the complexity of a simplified model for locating BSs in a W-CDMA network is studied for the downlink direction. However, the Polynomial Time Approximation Scheme presented therein is not suited to actual UMTS cellular systems since the model does only consider the interference between signals emitted by the same BS while neglecting the crucial aspect of the interference among BSs.

In this paper we propose discrete optimization models for planning BS location in UMTS networks, and describe randomized heuristics for finding good approximate solutions. Although these combinatorial optimization problems fall within the class of capacitated facility location problems [9], they differ from the classical ones from at least two points of view.

The paper is organized as follows. In Section 2 we discuss the main radio network planning issues pertaining to BS location and configuration for UMTS networks. In Section 3 we propose integer programming formulations for the W-CDMA setting in the uplink direction, which is expected to be the most stringent one from the traffic point of view in the presence of full-duplex balanced connections (e.g., voice calls). In Section 4 we describe three heuristics: randomized greedy and reverse greedy procedures as well as a randomized method that combines the two approaches in an adaptive way. Computational results are reported and discussed in Section 5. The last section contains some concluding remarks.

2. RADIO PLANNING ISSUES FOR W-CDMA

In UMTS networks the access scheme for the assigned paired spectrum (60 MHz from 1920 to 1980 MHz and 60 MHz from 2110 to 2170 MHz) is based on W-CDMA and frequency division duplexing, i.e, the two frequency bands are used separately for uplink and downlink. Prior to transmission, signals are spread over a wider band than that of the original signals using special codes. Codes used for signal transmitted in downlink by the same BS are mutually orthogonal, while those used for signals emitted by different stations (base or mobile) can be considered as pseudo-random. In an ideal environment, the de-spreading process performed at the receiving end can completely avoid the interference of orthogonal signals and reduce that of the others by the *spreading factor* (SF), which is the ratio between the spread signal rate and the user rate. In wireless environments, due to multipath propagation the interference among orthogonal signals is not equal to zero and the SIR in downlink is given by:

$$SIR_{downlink} = SF \, \frac{P_{received}}{\alpha I_{in} + I_{out} + \eta} \qquad (1)$$

where $P_{received}$ is the received power of the signal, I_{in} is the total interference due to the signals transmitted by the same BS, I_{out} that due to signals of the other BSs, α is the orthogonality loss factor ($0 \leq \alpha \leq 1$), and η the thermal noise power. In the uplink case, no orthogonality can be assumed. Therefore $\alpha = 1$ and we have:

$$SIR_{uplink} = SF \, \frac{P_{received}}{I_{in} + I_{out} + \eta} \qquad (2)$$

The quality of the received signal, usually expressed in terms of *bit error rate* (BER), mainly depends on the SIR, so it is common to consider quality constraints requiring that the SIR exceeds a minimum value τ which may vary according to the communication service considered (voice, video, data packets, etc.). For the sake of simplicity, in the sequel we refer to the minimum SIR before de-spreading as $SIR_{min} = \tau / SF$.

As mentioned above, the SIR depends on the received powers of the considered signal and the interfering ones. These in turn depend on the transmitted powers and the attenuation of the radio link between sources and destination. According to propagation conditions, the transmitted power can be adjusted to minimize interference and guarantee quality. So, for example, mobile stations close to the BS can reduce transmission power still guaranteeing a good quality but reducing the generated interference. These principles motivate the introduction of the PC mechanism. Dynamic PC mechanisms are key issues of all cellular systems and in particular of W-CDMA ones. Two PC methods are commonly considered, one based on the received power and one based on the estimated SIR. In the former one, the transmitted power is adjusted so that the power received on each channel is equal to a given target value P_{tar}. Similarly, in the second one, the transmitted power is set so that the SIR is equal to a target value SIR_{tar}. The latter mechanism is more complex, since the power emitted by each station depends on that emitted by all the others, but it allows the use of lower power [19].

Both in the case of received power-based or SIR-based PC, the transmitted powers are adjusted considering some power limit constraints. In particular, a limit on the maximum power used for each radio channel must be considered both for uplink and downlink. In the downlink case, an additional constraint is required on the total power emitted by each BS. Therefore, the actual power emitted on a channel is the minimum between that provided by the PC mechanism and the maximum value.

From a planning point of view, the effect of the power constraints is to limit the capacity of the system. Indeed in the receiver power-based PC, as new mobile stations are added to the system, the SIR values of all the others are decreased until the quality limit is reached on some connection and no more user can be safely added. On the other hand, in the SIR-based PC, the SIR is kept at a target value $SIR_{tar} \geq SIR_{min}$ and the quality is guaranteed. But, as new mobile stations are added, the power required to keep the SIR constant increases until the power limit is reached and the SIR falls below the minimum value.

During the network planning phase, a set of sites where it is possible to install BSs is provided. For each site a set of possible configurations, usually characterized by antenna direction and maximum emission power, is considered and for each of them the propagation conditions in the service area are taken as input of the planning phase (this information is usually obtained using propagation tools based on ray tracing or approximate radio channel models). Another input of the planning phase is the traffic distribution in the service area which is usually obtained with empirical prediction models. At the end of the planning phase we have to select the candidate sites where to install BSs and their configuration so that users can be assigned to BSs in a way that the minimum SIR is guaranteed on most (possibly all) connections.

Different planning objectives may be considered. If an installation cost is known for each BS, one may aim at minimizing the total cost while guaranteeing a full traffic coverage. This is conjectured to be the case for real-world instances where traffic patterns are based on short-to-mid term predictions and it is likely that the system will be required to serve all traffic. On the other hand, when traffic patterns are based on long term predictions a multi-period network planning strategy can be adopted and in the first stages one has to cope with solutions that do not cover all traffic. In this case, the focus is on maximizing the fraction of traffic demand that is actually satisfied. Finally, the minimization of emitted powers can also be included among the planning objectives in order to take into account electromagnetic pollution.

3. BASE STATION LOCATION PROBLEM WITH UPLINK CONSTRAINTS

According to the above observations, the problem of planning an UMTS network appears as a variant of classical *facility location* problems. Besides this common underlying structure, the integer programming models presented here and in [1] differ depending on the transmission direction (uplink or downlink) and the specific type of power control that are considered. The main focus of this paper is on the location problem with uplink constraints assuming a power-based PC mechanism. The extension of the model to SIR-based PC is also considered, while models for the downlink direction are presented in [1]. Note that in the uplink case the maximum emission powers of the various BSs play no role and our models can be easily extended to take into account a small number of different BS configurations.

Consider a territory to be covered by an UMTS service. Assume that a set of *candidate sites* $S = \{1, \dots, m\}$ where a BS can be installed, is given and that an installation cost c_j is associated with each candidate site j, $j \in S$. A set of *test points* (TPs) $I = \{1, \dots, n\}$ is also given. Each TP $i \in I$ can be considered as a centroid where a given amount of traffic d_i (in Erlang) is requested and where a certain level of service (measured in terms of SIR) must be guaranteed [17]. The required number of simultaneously active connections for TP i, denoted by u_i, turns out to be a function of the traffic demand, i.e., $u_i = \phi(d_i)$. The actual definition of the function ϕ is a degree of freedom of the planning process. It can simply correspond to the average number of active connections or to the number of simultaneous connections not exceeded with a given probability p. The connection activity factor can be considered as well.

The propagation information is also supposed to be known. In particular let g_{ij}, $0 < g_{ij} < 1$, be the propagation factor of the radio link between TP i, $1 \leq i \leq n$, and a candidate site j, $1 \leq j \leq m$. The propagation gain matrix $G = \{g_{ij}\}_{1 \leq i \leq n, 1 \leq j \leq m}$ is estimated according to approximate propagation models such as those proposed by Hata or to more precise but computationally intensive ray tracing techniques (see e.g. [12]).

In the W-CDMA UMTS base station location problem one wishes to select a subset of candidate sites within the set S where to install BSs, and to assign the TPs to the available BSs taking into account the traffic demand, the quality requirements in terms of SIR and the installation costs. This problem falls within the class of discrete location problems.

Let us define the two following classes of decision variables:

$$y_j = \begin{cases} 1 & \text{if a BS is installed in } j \\ 0 & \text{otherwise} \end{cases}$$

for $j \in S$ and

$$x_{ij} = \begin{cases} 1 & \text{test point } i \text{ is assigned to BS } j \\ 0 & \text{otherwise} \end{cases}$$

for $i \in I$ and $j \in S$. The core of the basic integer linear programming model that we propose for the uplink case is the classical uncapacitated facility location model:

$$\min \quad \sum_{j=1}^{m} c_j \, y_j + \lambda \sum_{i=1}^{n} \sum_{j=1}^{m} u_i \frac{P_{tar}}{g_{ij}} x_{ij} \tag{3}$$

$$s.t. \quad \sum_{j=1}^{m} x_{ij} = 1 \qquad i \in I \tag{4}$$

$$x_{ij} \leq y_j \qquad i \in I; j \in S \tag{5}$$

$$x_{ij}, y_j \in \{0, 1\} \qquad i \in I; j \in S. \tag{6}$$

While the first term in the objective function corresponds to the total installation cost, the second one favors assignments of the TPs to the BSs with smaller total power and the actual motivation is explained below. $\lambda \geq 0$ is a trade-off parameter. Constraints (4) make sure that each TP i is assigned to a single BS. Constraints (5) impose that TPs are only assigned to sites where a BS is installed.

The additional peculiarities of the uplink case to be taken into account are the quality of the signal received by each BS and the maximum emission power P_{max} of each mobile station. The constraint

$$\frac{P_{tar}}{g_{ij}} x_{ij} \leq P_{max} \, y_j \tag{7}$$

must be added for each pair of TP $i \in I$ and candidate site $j \in S$. Indeed, due to the power-based PC mechanism, P_{tar}/g_{ij} corresponds to the emission power from mobile station i required to guarantee the target signal power level P_{tar} in site j. Note that if $g_{ij} \, P_{max}/P_{tar} < 1$ the TP i cannot be assigned to candidate site j due to power limits, and therefore the variable x_{ij} can be omitted from the model. Otherwise constraint (7) is implied by the corresponding constraint (5).

As far as the signal quality is concerned, the use of pseudo-random spreading codes implies that, for a specific connection between TP i and BS j, there is no significant difference in the interference due to the signals emitted from the other TPs assigned to BS j (intra-cell interference) or from those assigned to different BSs (inter-cell interference).

In the presence of a power-based PC mechanism, for each candidate site $j \in S$ we consider the following quality constraint:

$$\frac{P_{tar}}{\sum_{h=1}^{n} u_h\, g_{hj} \sum_{t=1}^{m} \frac{P_{tar}}{g_{ht}}\, x_{ht} - P_{tar}} \geq SIR_{min}\, y_j, \qquad (8)$$

where the thermal noise η has been omitted for the sake of simplicity. This constraint enforces that if a BS is installed in site j (i.e., $y_j = 1$) the resulting SIR must exceed the given SIR_{min}. More specifically, the double summation in the denominator corresponds to the total interference due to all connections and the P_{tar} term that is subtracted amounts to the received power associated to a single connection assigned to j. Note that for any TP h the only term of the inner summation (over index t) that is nonzero corresponds to the site to which h is assigned. Thus the contribution to the outer summation of any TP h assigned to site j amounts to $u_h P_{tar}$. The bilinear constraints (8) can be linearized as follows:

$$1 + M(1 - y_j) \geq SIR_{min} \cdot \left(\sum_{h=1}^{n} u_h\, g_{hj} \sum_{t=1}^{m} \frac{1}{g_{ht}}\, x_{ht} - 1 \right) \qquad (9)$$

for a large enough value of M. It is worth emphasizing that constraints (9) are of the form:

$$\sum_{h=1}^{n} \sum_{t=1}^{m} a_{ht}^{j} x_{ht} \leq A - B y_j \quad j = 1, \ldots, m, \qquad (10)$$

where A and B are constants, and the coefficients a_{ht}^{j} depend on the candidate site j under consideration and also on the particular BS t the TP h is assigned to. Due to the values of A and B, constraints (10) are always satisfied when $y_j = 0$ regardless of the way the TPs are assigned to the BSs, and they amount to

$$\sum_{h=1}^{n} \sum_{t=1}^{m} a_{ht}^{j} x_{ht} \leq \frac{1}{SIR_{min}} \qquad (11)$$

when $y_j = 1$. This is clearly in contrast with standard capacity constraints arising in capacitated facility location problems that can be expressed as:

$$\sum_{h=1}^{n} d_h x_{hj} \leq s_j y_j \quad j = 1, \ldots, m, \qquad (12)$$

where the "demand" d_h of "client" h does not depend on the "facility" to which "client" h is assigned (there is no summation on the second subscript of the x variables) and, for each facility j, the constraint only involves the "clients" assigned to it. In the UMTS BS location problem, it is as if each TP h absorbs part of the capacity of every BS and not only that of the BS it is assigned to. Moreover, the amount requested from BS j depends on the "distance" (gain) between TP h and BS j as well as on the "distance" (gain) between TP h and the BS it is assigned to.

Note that by restricting the assignment variables x_{ij} to take binary values, it is required that in every feasible solution all active connections out of any TP must be assigned to a single BS.

The following result establishes a simple but important property of the underlying assignment subproblem of the BS location problem. For any TP i in I and candidate site j in S let $p_{ij} = \frac{P_{tar}}{g_{ij}}$ denote the emission power needed to guarantee a signal of power P_{tar} at site j. Moreover, for any given assignment X of TPs to BSs, let us denote by $SIR_X(j)$, $j \in S$, the actual SIR value at site j, which is defined as the left hand side term of (8).

Proposition *Given any assignment X of TPs to a set of active BSs $\overline{S} \subseteq S$ which yields SIR values $SIR_X(j)$ for all $j \in \overline{S}$, any assignment $\tilde{X}$ in which each TP $i \in I$ is assigned to a "closest" active BS in $\overline{S}$, i.e., to one with the smallest p_{ij} over all $j \in \overline{S}$, gives SIR values such that $SIR_{\tilde{X}}(j) \geq SIR_X(j)$ for all $j \in \overline{S}$.*

Proof Consider any assignment X of TPs to the active BSs $\overline{S} \subseteq S$ and assume that at least one TP, say $h \in I$, is not assigned to a BS $t \in \overline{S}$ with $p_{ht} = \min_{j \in \overline{S}} p_{hj}$. According to the definition of the required emission power p_{ht}, a smaller p_{ht} corresponds to a larger g_{ht} over all $t \in \overline{S}$. It is easily verified that if h is now re-assigned to a BS t with the largest g_{ht} the denominator of each constraint (8), for $j \in \overline{S}$, decreases and, equivalently, the right hand side of each constraint (9) increases. Therefore the new assignment X' in which TP h is assigned to a "closest" active BSs triggers a set of SIR values, $SIR_{X'}(j)$, such that $SIR_{X'}(j) \geq SIR_X(j)$ for all $j \in \overline{S}$. The result is then obtained by applying this re-assignment step to all TPs that are not assigned in X to a "closest" BS.

According to this proposition, only assignments of TPs to "closest" active BSs need to be considered. This makes the problem under consideration substantially different from standard capacitated facility location problems where, for any given set of active BSs, the optimal assignment is not known a priori and needs to be determined.

This aspect, which can be accounted for explicitly in the model by adding the following constraint:

$$\sum_{j=1}^{m} g_{ij}\, x_{ij} \geq g_{it}\, y_t \qquad (13)$$

for each $i \in I$ and $t \in S$, is also partially taken into account by the second term of the objective function (3).

Clearly, depending on the traffic requirements and distribution, the number of candidate sites and locations as well as the mobile station maximum power, some of constraints (8) can make the model infeasible. This can be dealt with either by relaxing these constraints and including them in some way in the objective function, or by minimizing the number of TPs whose traffic demand cannot be satisfied. In the former case, one may simply add to the objective function the sum of deviations from satisfiability of constraints (8) over all candidate sites. In the latter one, the assignment constraints (4) can be relaxed as follows:

$$\sum_{j=1}^{m} x_{ij} \leq 1 \qquad i \in I \tag{14}$$

so as to allow some TPs not to be assigned. To look for solutions satisfying a maximum fraction of the traffic demand, the additional term $\sum_{i\in I} u_i \sum_{j\in S} x_{ij}$ can be added to the objective function (to be minimized) with an appropriate negative weight parameter.

In the presence of the more sophisticated PC mechanism aiming at constant estimated SIR, the discrete optimization model is more involved since the signal powers must be considered as variables. For each TP i, we define p_i as the emission power needed to guarantee an estimated SIR greater or equal to SIR_{tar} at the site (BS) to which TP i is assigned. The core model is then as in (3-6) except that in the second term of the objective function P_{tar}/g_{ij} is replaced by p_i. Constraints (7) are replaced by $0 \leq p_i x_{ij} \leq P_{max} y_j$. Moreover, for each pair of TP $i \in I$ and candidate site $j \in S$, the quality constraint is now:

$$\frac{p_i\, g_{ij}}{\sum_{h=1}^{n} u_h\, g_{hj} \sum_{t=1}^{m} p_h\, x_{ht} - p_i g_{ij}} \geq SIR_{tar}\, x_{ij} \tag{15}$$

where again the thermal noise η has been omitted. Note that this third order constraint enforces that if TP i is assigned to site j (i.e., $x_{ij} = 1$) the resulting SIR must exceed the given SIR_{tar}. Unlike in the power-based PC, for each BS j there is here a SIR constraint for each TP assigned to it, since the received power may differ for different connections.

4. RANDOMIZED HEURISTICS

Since the UMTS BS location problems defined in the previous section contain the uncapacitated facility location problem as a special case, they turn out to be NP-hard. To obtain good approximate solutions within a reasonable amount of computing time, we have developed greedy (see [3]) and reverse greedy randomized procedures that iteratively activate and, respectively, disactivate BSs as well as a randomized method which combines the two above types of steps. The focus here is on the power-based PC mechanism.

According to the previous section, by assigning each TP to a "closest" active BS we are guaranteed to obtain an optimal solution from the SIR value point of view. However, since it is not known a priori whether all traffic demands can be satisfied, the central procedure of our greedy and reverse greedy heuristics aims, given a current set of active BSs, at satisfying the largest fraction of traffic demand (see the relaxed constraints (14)). For each TP i in I the smallest emission power needed to guarantee a signal of power P_{tar} at some of active BSs is computed, namely, we define

$$P_i = \min_{j \in \overline{S}} \frac{P_{tar}}{g_{ij}} \tag{16}$$

where $\overline{S} \subseteq S$ denotes the current set of active BSs. Let $\overline{s}_i \in \overline{S}$ denote the index of an active BS which provides the minimum in (16). Given any subset $\overline{S} \subseteq S$, the subproblem of selecting a maximum number of TPs to be served so as to satisfy the SIR constraints (9) for all active BSs ($j \in \overline{S}$) can be restated as the following selection problem:

$$\begin{array}{lll} \max & \sum_{h\in I} u_h\, z_h & \\ s.t. & \sum_{h\in I} u_h \frac{g_{hj}}{g_{h\overline{s}_h}} z_h \leq \frac{1}{SIR_{min}} & j \in \overline{S} \\ & z_h \in \{0,1\} \quad h \in I & \end{array} \tag{17}$$

where $z_h = 1$ if TP h is selected (and therefore assigned to $\overline{s}_h$) and 0 if it is discarded, namely, not served. The above subproblem amounts to a special case of multi-knapsack problem which turns out to be hard to solve optimally. Since in our context we need reasonably good solutions in a short amount of time we proceed as follows. TPs can be considered by nondecreasing value of P_i/u_i and are assigned to the corresponding "closest" BS if the SIR values for all BSs still exceeds SIR_{min}. Whenever a SIR constraint of any BS is not satisfied, the TP is discarded. The complexity of the this procedure is $O(n \log(n) + n\, m' + n^2)$, where n and $m' = |\overline{S}|$ are the number of TPs and of active BSs, respectively. Procedures that better exploit the structure of the multi-knapsack problem and involve upper bounds obtained either by linear programming or surrogate relaxation can also be used. In the former case z_h, which takes values between 0 and 1, can be viewed as the fraction of the traffic demand u_h that is actually serviced. In the latter one, by aggregating all the constraints into a single one, one obtains an instance of the well-studied knapsack problem. However in both cases this considerably increases the computational time required to solve the selection subproblem that must be tackled at each step of our algorithms.

In the first randomized greedy algorithm (Add), BSs are added iteratively to $\overline{S}$ starting from $\overline{S} = \emptyset$. Given the current set $\overline{S} \subseteq S$ of sites in which a BS has already been installed, for each remaining candidate site $j \in S \setminus \overline{S}$ the above procedure is applied to $\overline{S}' = \overline{S} \cup \{j\}$ so as to obtain a corresponding assignment vector $\mathbf{z}$. Each pair $(\overline{S}, \mathbf{z})$ fully characterizes a solution of the original problem since it implies the values of all x and y variables. For each of these solutions $\overline{S}'$, the following utility function is evaluated:

$$U_a(\overline{S}, j) = \sum_{h=1}^{n} u_h\, z_h - \omega \sum_{t=1}^{m} c_t y_t, \tag{18}$$

where the first term amounts to the fraction of traffic that is currently covered and the second one expresses the total cost for installing the BSs in the sites selected so far. The trade-off parameter ω, $\omega > 0$, allows us to assign higher priority to maximize the first objective than to minimize the second one. At each iteration one $j' \in S \setminus \overline{S}$ is randomly selected among the $\rho_a \geq 1$ ones that yield the largest value of U_a. The procedure stops when the addition of a new BS worsen the current solution value according to utility function U_a.

In the randomized reverse greedy algorithm (Remove), BSs are removed iteratively starting from $\overline{S} = S$. Given the current $\overline{S} \subseteq S$, for each candidate site $j \in \overline{S}$ the above procedure is applied to $\overline{S} \setminus \{j\}$ so as to obtain a corresponding $\mathbf{z}$. Then the following utility function is evaluated:

$$U_r(\overline{S}, j) = \sum_{h=1}^{n} u_h\, z_h - \omega \sum_{t=1}^{m} c_t y_t + \nu F(\mathbf{z}), \tag{19}$$

where $F(\mathbf{z})$ is the sum, over all BSs j that have so far been installed, of the number of additional connections they could service, namely, of $\lfloor \Delta_j / P_{tar} \rfloor$ where Δ_j is defined as the difference between the current SIR and SIR_{min}. $\nu > 0$ is the corresponding weight. At each iteration one $j' \in \overline{S}$ is randomly selected among the $\rho_r \geq 1$ ones that yield the largest value of U_r. As for the Add procedure, the Remove procedure stops when the removal of another BS worsen the current solution value according to utility function U_r.

Given the randomized nature of the Add and Remove procedures and their relatively low computational requirements, a multistart strategy is adopted: they are run a predefined number of times and provide as output the best solution found.

A third heuristic, referred to as the RAR (Randomized Add and Remove) algorithm, is obtained by combining the two types of moves of the above procedures. In particular, we define as an *Add move* the operation of activating a new BS, and a *Remove move* that of turning one of the currently activated BSs off. Our algorithm can be viewed as a variant of "simulated allocation" that has been proposed in [13] for optimal design of optical networks. The iterative procedure starts with a randomly generated solution and at each iteration either an Add or a Remove move is applied. An Add move is carried out according to a given probability π, $0 \leq \pi \leq 1$, and a Remove move with probability $1 - \pi$. Whenever a move improves the objective function value, the probability of applying again the same type of move is increased by a given factor δ. More precisely, if an Add move yields an improved solution in terms of the first utility function U_a, the value of π is increased to $\pi + (1 - \pi)\delta$. In case the objective function value is not improved, π is decreased to $\pi(1 - \delta)$. When Remove moves are carried out the probability $1 - \pi$ is updated in the same way but considering the second utility function U_r. The procedure also includes a *diversification* strategy: if no improvement occurred during a prescribed number of iterations, a new starting solution is randomly generated by turning on or off a fraction of active BSs and the probability π is reset to 0.5

5. COMPUTATIONAL RESULTS

To evaluate the performance of the proposed algorithms we have considered synthetic but realistic uplink instances generated by using the Hata's propagation model [8]. For each instance we consider a rectangular service area with dimensions $L \times W$, a number m of candidate sites in which to locate omnidirectional antennas and a number n of TPs. Using a pseudo-random number generator each candidate site j and each TP i is assigned a position with uniform distribution in the service area. The matrix $\mathbf{G}$ is obtained by using the Hata's formulas which give the attenuation A (loss) in dB due to signal propagation. In particular, the attenuation for urban areas is given by:

$$\begin{aligned} A_u \;=\; & 69.55 + 26.16 \log(f) - 13.82 \log(Hb) + \\ & - [(1.1 \log(f) - 0.7)Hm - (1.56 \log(f) - 0.8)] + \\ & + [44.9 - 6.55 \log(Hb)] \log d, \end{aligned} \tag{20}$$

where f is the signal frequency in MHz, Hb and Hm are the heights of the base and the mobile station in meters, and d is the distance in Km, while the formula for rural areas is:

$$A_r = A_u - 4.78[\log(f)]^2 + 18.33 \log(f) - 35.94. \tag{21}$$

Clearly $g_{ij} = 1/10^{A(d_{ij})/10}$, where d_{ij} denotes the distance between TP i and candidate site j.

We considered three families of uplink instances with a service area of 400 × 400 m, 1 × 1 Km and 1.5 × 1.5 Km respectively. Different instances of these families are obtained using different seeds for the pseudo-random generator. Two gain matrices $\mathbf{G}$ are considered for each configuration using the urban and the rural Hata's formulas with:

$$Hb = 10 \ \ m, \qquad Hm = 1 \ \ m, \qquad f = 2000 \ \ MHz.$$

To complete the instance definition we used identical installation costs $c_j = c$ for all $j \in S$, $SF = 128$ and $\tau = 4$ (so $SIR_{\text{min}} = 0.03125$), $P_{\text{max}} = 30$ dBm, $P_{\text{tar}} = -100$ dBm. In all the experiments reported here $\omega = 1/(c\ m)$ and $\nu = 1/m^2$.

The results for small instances with a service area of 400 × 400 meters, $m = 22$ candidate sites, $n = 95$ test points, and $u_i = 1$ for all TPs $i \in I$, obtained with the Add and Remove algorithms ($\rho_a = \rho_r = 4$) over 50 runs are reported in Table 1. The solutions found always satisfy all 95 requested connections. Out of the 10 instances mentioned in Table 1, 5 have been generated using the Hata formula for urban areas (SU) and 5 using that for rural areas (SR). For the same instances the optimal solutions, involving 4 BSs in all cases, have been obtained by solving the integer linear programming formulation with a state-of-the-art commercial solver (CPLEX). In most of the cases the Add and Remove algorithms provide the optimal solution and the number of BSs installed never exceeds the minimum number by more than 1. It is worth noting that the computation time for Add and Remove was less than 5 seconds on a Pentium III/700 MHz PC, while CPLEX required 5 to 20 minutes to solve each instance. As a matter of fact, solving even such small instances using CPLEX turns out to be a challenging task due to the very delicate choice of the value M. If the parameter M is too small, then some SIR constraints that should be omitted will not be disactivated and, therefore, the resulting solutions contain a larger number of BSs than optimal solutions of the actual problem. On the other hand, for too large values of M, one runs into numerical problems related to machine precision which tend to override some SIR constraints and, hence, to yield solutions with a too small number of BSs.

	Add			Remove		
SU-1	4	5.96	0.86	4	5.14	0.40
SU-2	4	5.42	0.57	4	4.62	0.49
SU-3	4	5.84	0.81	4	4.68	0.51
SU-4	5	5.28	0.45	5	6.92	0.98
SU-5	5	5.38	0.52	4	4.82	0.43
SR-1	4	5.04	0.60	4	4.90	0.46
SR-2	4	5.44	0.67	4	4.74	0.44
SR-3	4	5.42	0.64	4	5.04	0.49
SR-4	5	6.68	0.86	5	5.32	0.55
SR-5	4	5.12	0.47	4	4.96	0.34

Table 1: Results obtained with the Add and Remove algorithms for a set of small-size urban (SU) and rural (SR) instances characterized by $L = W = 400$ meters, $m = 22$, $n = 95$, $u_i = 1$. For each algorithm, from left to right: minimum number of BSs installed, the average number and the standard deviation over 50 runs.

Table 2 shows the results obtained for these small instances with the RAR algorithm. A maximum number of 1200 moves has been considered. $\delta = 0.2$ and the diversification strategy was triggered when more than 6 moves within a sliding window of 11 were not improving. The close-to-optimal solutions found are obtained within the first one hundred moves.

	RAR	
SU-1	4	76
SU-2	4	71
SU-3	4	86
SU-4	5	14
SU-5	4	26
SR-1	4	106
SR-2	4	62
SR-3	4	76
SR-4	5	31
SR-5	4	97

Table 2: Results of the RAR algorithm for the small instances of Table 1. From left to right: number of BSs installed and step in which the best solution has been obtained.

Table 3 and 4 reports the results obtained with the Add and Remove algorithms (50 runs) and, respectively, with the RAR algorithm for medium-size instances with a service area of 1×1 Km, $m = 120$, $n = 400$, and u_i geometrically distributed with mean equal to 2, for all $i \in I$. The solutions found always provided full traffic coverage.

	Add			Remove		
MU-1	50	50.62	1.63	51	52.96	0.66
MU-2	46	47.76	1.38	46	47.84	4.36
MU-3	43	45.28	1.15	44	47.70	3.80
MU-4	45	47.32	1.65	46	49.06	4.35
MU-5	43	46.24	1.28	45	49.44	3.12
MR-1	44	45.12	0.79	42	46.60	2.16
MR-2	46	47.76	1.38	46	47.16	0.88
MR-3	43	45.28	1.13	44	48.12	3.78
MR-4	43	45.72	1.47	45	48.44	4.11
MR-5	44	46.38	1.13	46	50.48	3.68

Table 3: Results obtained with the Add and Remove algorithms for a set of medium-size urban (MU) and rural (MR) instances characterized by $L = W = 1$ Km, $m = 120$, $n = 400$, u_i geometrically distributed with mean equal to 2. For each algorithm, from left to right: minimum number of BSs installed, the average number and the standard deviation over 50 runs.

The number of BSs installed by our Add and Remove algorithms ($\rho_a = \rho_r = 4$) differ at most by 2. Add yields solutions that are at least as good as the one provided by Remove for all but one instances. Moreover, the standard deviation of the number of BSs in the solutions given by Remove (over the 50 runs) is usually greater than that of the solutions provided by Add. The 50 runs of Add and the 1200 moves of RAR took in average 3600 to 3800 seconds. Note that if a small number of BSs needs to be installed, the Remove version is more time consuming. The same comment is of course valid for the Add version in the opposite situations where BSs must be installed in most of the candidate sites.

As shown in Table 4, the RAR algorithm ($\pi = 05$, $\delta = 0.2$) yields for all but two instances a solution better than the ones obtained with Add and Remove. In more than half of the cases these solutions are obtained within less than 500 moves. For the instances MU-5 and MR-4, the RAR solutions turn out to be a compromise between those yielded by Add and Remove.

	RAR	
MU-1	48	206
MU-2	44	108
MU-3	43	526
MU-4	44	625
MU-5	43	815
MR-1	42	822
MR-2	45	1016
MR-3	43	352
MR-4	43	232
MR-5	44	111

Table 4: Results of RAR algorithm for the medium-size instances of Table 3. From left to right: number of BSs installed and step in which the best solution has been obtained.

We have also carried out 50 runs of Add and Remove ($\rho_a = \rho_r = 10$) and 1400 moves of RAR on five large urban instances with a service area of 1.5×1.5 Km, $m = 200$, $n = 750$, and u_i geometrically distributed with mean equal to 2, for all $i \in I$. This choice of parameter values, with 1500 active connections in the average, is quite realistic for UMTS setting in medium-to-large cities. The results are reported in Table 5 and in Table 6. Unlike for the previous classes of instances, the traffic demand is not always satisfied. It is worth noting that Remove provides solutions of equal or better quality for all but the first instance.

	Add			Remove		
LU-1	70	70.58	1106/1107	87	132.14	1106/1107
LU-2	74	70.26	1135/1136	70	95.29	1136/1136
LU-3	66	69.00	1120/1124	61	73.54	1120/1124
LU-4	66	69.58	1130/1130	66	90.26	1130/1130
LU-5	64	67.36	1103/1103	59	66.36	1103/1103

Table 5: Results for a set of large urban instances characterized by $L = W = 1.5$ Km, $m = 200$, $n = 750$, u_i geometrically distributed with mean equal to 2. From left to right: minimum number of BSs installed, average number over 50 runs, and fraction of traffic covered.

	RAR		
LU-1	64	158	1106/1107
LU-2	68	1259	1136/1136
LU-3	61	1261	1120/1124
LU-4	61	369	1130/1130
LU-5	58	202	1103/1103

Table 6: Results of RAR algorithm for the large instances of Table 5. From left to right: number of BSs installed, step in which the best solution has been obtained and fraction of traffic covered.

According to Table 6, the RAR algorithm ($\pi = 0.5$, $\delta = 0.2$) always yields solutions of better or equal quality than the best ones provided by Add and Remove. These solutions do also cover the same fraction of traffic. Note that for some instances, like LU-1 and LU-4, the improvement in terms of BSs installed is substantial.

6. CONCLUDING REMARKS

We have first discussed the main features characterizing the important problem of planning UMTS networks using the W-CDMA radio access technique. Focusing on the uplink direction, we have proposed integer programming formulations for the UMTS BS location problem with power-based as well as SIR-based PC mechanisms. Our randomized greedy and reverse greedy heuristics and the combined Randomized Add and Remove algorithm provide, in a reasonable amount of time, good approximate solutions for medium-to-large size realistic uplink instances. These solutions can be further improved by some local search steps.

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