Joint Optical Network Design, Routing and Wavelength Assignment by Integer Programming*

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Abstract

We present a new mathematical model for all-optical network design, including sparse optical cross connects placement, traffic routing and wavelength assignment. The proposed linear integer program is refined by introducing valid inequalities, and a cutting plane procedure is described. The solution procedure is implemented using commercial mixed integer programming solvers and applied to some real instances of metropolitan and wide area networks. We present encouraging results that show the validity of the approach.

Keywords: Network design, WDM, OXC placement, Integer Programming.

1 Introduction

The telecommunication industry is currently moving towards the massive use of optical networks. To increase the capacity of the existing optical network infrastructure, *Wavelength Division Multiplexing* (WDM) or *Dense Wavelength Division Multiplexing* (DWDM) devices are introduced. These devices are used to send multiple data streams in the same fiber, and appear as an interesting alternative to installing new fiber cables. In WDM technology, light signals with different wavelengths (colors) carry multiple data streams in one fiber. Since WDM has been mainly used on point-to-point links, a double-conversion (opto–electrical and electro–optical) is required each time a data stream passes a network node. In order to improve the efficiency and the reliability of the network, optical switches may be introduced at the nodes, so that the conversions are avoided

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and the so called "transparency" of the network is achieved ([11]). Simple optical switches allow an incoming data stream with a certain wavelength in a fiber to be forwarded to another fiber using the same wavelength. However, there is also another type of optical switch, known as *Optical Cross Connects* (OXCs). An OXC can convert any data stream entering a node with a given wavelength into a stream leaving the node with any wavelength. This wavelength switching is carried out purely at the optical level without the need of any transition to the electrical level, thus avoiding delays and possible loss of data. The reason for switching the wavelength used by a data stream is to ensure that a wavelength is assigned to at most one data stream in any fiber. As OXC technology is rather new, it is currently very expensive and therefore OXCs should only be installed at the nodes where wavelength switching is strictly necessary.

Without OXCs, the wavelength assignment problem can be described as follows. Given a directed graph G and a set of path P in G, we wish to assign a color to each path in P, such that any two paths sharing one or more arcs are assigned different colors. Placing an OXC at a node v enables a path containing v to change the color at v. In a network with OXC nodes, we wish to, for every path, assign a color to each of its arcs, with the restriction that the color of a path can change only when the path passes through an OXC node.

Typically, the problem of routing and wavelength assignment as well as the problem of network design and capacity dimensioning, have been tackled separately in the literature. Two versions of the problem have previously been studied for routing and wavelength assignment. In the first version, it is assumed that wavelength switching can take place at all network nodes, hence an OXC is required at each node. In the second version, wavelength switching is not allowed at all and hence no OXCs are needed. Both problems are $\mathcal{NP}$–hard. The first problem is equivalent to an integer multicommodity flow model, and a reduction from graph coloring has been provided in [7] for the second problem. Several mathematical formulations and heuristic algorithms have been presented for both problems (see for example [3], [14], and [15]). Recently, the problem of solving optical network design, routing, and wavelength assignment simultaneously has been studied in [13] and [6]. The two papers provide heuristic algorithms which either assume that an OXC is already present at each node, or assume that no OXC is used in the network. An extended review of the literature on this subject can be found in [9].

In this paper, we consider the optical network planning problem where an all-optical DWDM network is to be designed, or where an existing network is to be upgraded with DWDM technology. Given the traffic matrix and the costs involved in network planning (including the costs of installing OXCs and wavelength multiplexers), the optimization problem consists of determining the traffic routing, the capacity of each link, and the placement of the OXCs as well as the wavelength assignment, such that the total cost is minimized.

Since DWDM technology is relatively new, the literature does not present extensive studies or clear indications on how to deal with this kind of network planning problem when the decisions involve where to place OXCs. In the recent survey into optical networks provided in [9], the joint optimization problem of optical network design, routing, wavelength assignment and OXC installation is considered as one of the most challenging problems in this field. Common approaches for tackling practical applications adopt a two-step procedure (see e.g. [8] and [2]). In the first step, a subset of possible routes for each pair of nodes in the traffic matrix is defined. Usually these routes are generated by finding the k–shortest paths from the origins to the destinations.

Then in the second step, it is determined at which nodes the OXCs are to be installed, and how many new fibers are to be placed in each link.

We propose a method for approaching the joint problem of network design, traffic routing, wavelength assignment and OXC installation in one step. In Section 2, we present a mathematical formulation of the problem that is based on an *integer multicommodity flow* model. We then introduce some classes of valid inequalities to the model by exploiting the combinatorial properties of the network in Section 3. A cutting plane algorithm is developed and implemented using a modeling language and a standard Mixed Integer Programming (MIP) solver. Numerical results of some metropolitan and of wide area network instances are reported and analyzed in Section 4. The results show that the cutting plane procedure is efficient in providing good lower bounds. In addition, optimal solutions can be found efficiently by combining the cutting plane procedure with an MIP solver. We finally draw some conclusions and give suggestions for future work in Section 5.

2 Notation and problem definition

We consider a network represented by a graph $G = (N, A)$, where N is the set of *network nodes* at which OXC equipment can be installed, and A is the set of *potential arcs*. An arc $(u, v) \in A$ implies that a fiber may be installed from u to v. The capacity of an arc can be chosen among values in a discrete set, and is measured by the number of wavelengths. If no WDM is used for an arc, the capacity is one, otherwise the capacity will depend on the type of WDM technology used. The mathematical model in this section considers the case where an arc represents a single fiber. However, extending the model to cover the case of multiple fibers per arc by adding another index for different fibers for each arc is straightforward. Let $K = \{1, \ldots, |K|\}$ be a set of *commodities*. A commodity k is characterized by a pair of nodes: s^k defines the *origin* and t^k defines the *destination*. We use d^k to denote the *demand* of commodity k; that is the amount of traffic, in the number of wavelengths, to be routed from s^k to t^k. The demands of the commodities in K form a *traffic matrix* of the network.

A *path* of commodity k in G, is specified by a sequence of consecutive arcs from the origin s^k to the destination t^k. It is assumed that a set of *feasible paths*, P^k, is given for commodity k. The criteria used to determine the feasibility of a path may include the physical length and the number of arcs, as well as other considerations that are related to signal attenuation. Let P denote the *set of feasible paths* for all commodities, i.e. $P = \cup_{k \in K} P^k$. We also introduce the notation $P(u, v)$, which is the set of paths in P that contain arc (u, v).

The problem formulation of optical network design, routing and wavelength assignment depends on whether the demand of a commodity must follow the same path from the origin to the destination, or if it can be routed along several paths. In this paper, we consider the case in which the entire demand for a commodity is to be routed along a single path, and define the following variables to represent the *path selection* for the commodities.

$$x_p = \begin{cases} 1 & \text{if a given commodity } k \text{ is routed on path } p \text{, for } p \in P^k; \\ 0 & \text{otherwise.} \end{cases}$$

Note that since any path in P is associated with a specific commodity, we do not need to have the commodity index for the path variables.

We also need to define variables for the wavelength assignment of each flow unit in the network. We assume that the maximum number of wavelengths available is the same for each arc, although both the mathematical model and the solution procedure can easily be extended to handle more general cases. We use $H = \{1, \ldots, |H|\}$ to denote the set of *available wavelengths*, and define the variables for wavelength assignment below.

$$y_{uv}^{kh} = \begin{cases} 1 & \text{if wavelength } h \in H \text{ is assigned to one unit of commodity } k \text{ on arc } (u,v); \\ 0 & \text{otherwise.} \end{cases}$$

We also introduce the following variables to model optical switching operations.

$$z_v^k = \begin{cases} 1 & \text{if a wavelength switching occurs for commodity } k \text{ at node } v; \\ 0 & \text{otherwise.} \end{cases}$$

The mathematical model for finding the optimal routing and wavelength assignment that minimizes the total number of switching operations is stated below.

$$\min \quad \sum_{v \in N} \sum_{k \in K} z_v^k \tag{1}$$

$$\sum_{p \in P^k} x_p = 1, \ \forall k \in K \tag{2}$$

$$\sum_{k \in K} y_{uv}^{kh} \leq 1, \ \forall h \in H, \forall (u,v) \in A \tag{3}$$

$$\sum_{h \in H} y_{uv}^{kh} = \sum_{p \in P^k \cap P(u,v)} d^k x_p, \ \forall k \in K, \forall (u,v) \in A \tag{4}$$

$$z_v^k \geq y_{wv}^{kh} - y_{vu}^{kh} - (1 - \sum_{p \in P^k \cap P(w,v) \cap P(v,u)} x_p) \tag{5}$$

$$\forall k \in K, \forall h \in H, \forall (w,v), (v,u) \in A$$

$$x_p \in \{0,1\}, \ \ \forall p \in P \tag{6}$$

$$y_{uv}^{kh} \in \{0,1\}, \ \ \forall h \in H, \forall k \in K, \forall (u,v) \in A \tag{7}$$

$$z_v^k \in \{0,1\}, \ \ \forall v \in N, \forall k \in K. \tag{8}$$

If we wish to minimize the number of OXCs instead of the number of switching operations in the network, the objective will be to minimize the number of nodes where any wavelength switching takes place. This can be done by using the following objective function instead of (1).

$$\min \sum_{v \in N} (\max_{k \in K} z_v^k). \tag{9}$$

Constraints (2) ensure that exactly one path is selected for each commodity, and constraints (4) state the relationship between the network capacity and the commodity demands. A constraint of (3) states that any wavelength can be used at most once along an arc. Observe that several wavelengths can be used, but they have to be assigned to different demand units. Constraints (5), together with the minimization of the objective function, define the values of variables z_v^k. It can be realized that $z_v^k = \max_{h \in H} |y_{wv}^{kh} - y_{vu}^{kh}|$ in any feasible solution, that is the value of variable z_v^k becomes one if and only if commodity k leaves node v with a different wavelength than the one it uses to reach v. For a more detailed discussion of this model and alternative models, see [5].

To introduce the variables and constraints for the arc capacities in the network, we assume that for each arc, the capacity can be chosen among a number of possible levels that form a discrete set. We assume that different types of WDM technology can be combined in a network therefore we may have different WDM technologies (i.e., different numbers of wavalengths) on different arcs. Then those capacity levels are related to the different types of WDM technology that are available, and are specified in the number of wavelengths. We use $L = \{1, \ldots, |L|\}$ to denote the *set of available capacity levels.* Let c_ℓ be the number of wavelengths associated with capacity level ℓ. We also assume that $c_1 = 1$, which corresponds to conventional fiber technology where WDM is not used. We introduce the following network design variables.

$$Y_{uv}^{\ell} = \begin{cases} 1 & \text{if capacity } c_\ell \text{ is installed on arc } (u,v); \\ 0 & \text{otherwise.} \end{cases}$$

We define the following constraints for the arc capacities.

$$\sum_{k \in K} \sum_{h \in H} y_{uv}^{kh} \leq \sum_{\ell \in L} c_\ell Y_{uv}^{\ell}, \ \forall (u,v) \in A \tag{10}$$

$$\sum_{\ell \in L} Y_{uv}^{\ell} \leq 1, \ \forall (u,v) \in A. \tag{11}$$

Constraints (10) ensure that if capacity level l is installed for an arc, a maximum of c_ℓ wavelengths can be used in the wavelength assignment. Constraints (11) state that no more than one capacity level can be chosen for any arc. Note that a feasible solution may have $Y_{uv}^{l} = 0, \forall l$, which means that no fiber is to be installed for arc (u,v), or that an already existing fiber is not used.

We use g_{uv}^{ℓ} to denote the *cost of installing a WDM device* providing capacity level ℓ. If a fiber already exists for (u,v), we assume that $g_{uv}^1 = 0$, and set $Y_{uv}^1 = 1$. Moreover, we let f_v denote the *cost of installing an OXC* in node v. The complete model for optical network design, routing, wavelength assignment, and OXC placement is stated below.

$$\min \quad \sum_{v \in N} f_v(\max_{k \in K} z_v^k) + \sum_{\ell \in L} \sum_{(u,v) \in A} g_{uv}^{\ell} Y_{uv}^{\ell} \tag{12}$$

$$\text{s.t.} \quad (2), (3), (4), (5), (6), (7), (8), (10), (11)$$

$$Y_{uv}^{\ell} \in \{0,1\}, \quad \forall \ell \in L, \forall (u,v) \in A. \tag{13}$$

2.1 An alternative formulation

We present an alternative problem formulation in which the capacity constraints are formulated in an incremental manner. This alternative formulation is equivalent to the first formulation stated above, but it allows us to generate valid inequalities very simply. Without loss of generality, we assume that for any arc $(u,v) \in A$, we have $c_{\ell-1} < c_\ell$ and $g_{uv}^{\ell-1} < g_{uv}^{\ell}$, for $\ell = 2,\ldots,|L|$. We also introduce an auxiliary capacity level $\ell = 0$, with $c_0 = 0$ and $g_{uv}^0 = 0$, and define the design variables as follows.

$$Y_{uv}^{\ell} = \begin{cases} 1 & \text{if capacity } (c_\ell - c_{\ell-1}) \text{ is added to arc } (u,v); \\ 0 & \text{otherwise.} \end{cases}$$

The entire model can now be formulated in the following way.

$$\begin{aligned} \min \quad & \sum_{v\in N} f_v(\max_{k\in K} z_v^k) + \sum_{\ell\in L}\sum_{(u,v)\in A}(g_{uv}^{\ell} - g_{uv}^{\ell-1})Y_{uv}^{\ell} && (14)\\ \text{s.t.} \quad & (2),(3),(4),(5),(6),(7),(8) \\ & \sum_{k\in K}\sum_{h\in H} y_{uv}^{kh} \le \sum_{\ell\in L}(c_\ell - c_{\ell-1})Y_{uv}^{\ell}, \ \forall (u,v)\in A && (15)\\ & Y_{uv}^{\ell} \le Y_{uv}^{\ell-1}, \ \ell = 2,\ldots,|L|, \forall (u,v)\in A && (16)\\ & Y_{uv}^{\ell} \in \{0,1\}, \ \ \forall \ell \in L, \forall (u,v)\in A. && (17) \end{aligned}$$

Note that in this model, $Y_{uv}^{\ell} = 1$ for some ℓ implies $Y_{uv}^{\ell'} = 1$ for all $\ell' < \ell$. It is obvious that the two problem formulations are equivalent.

3 Strengthening the model

Consider the second problem formulation described in Section 2.1. The relaxation of the integrality constraints (LP-relaxation), which is usually done in the Branch and Bound algorithms implemented by commercial MIP solvers, often provides lower bounds that are very poor (see also the computational results in Section 4). The study of valid inequalities that can improve the LP-relaxation is therefore motivated.

Consider arc (u,v) and let $K(u,v)$ be the set of commodities that may use this arc (i.e. $K(u,v) = \{k : P^k \cap P(u,v) \neq \emptyset\}$), and let ℓ be the capacity level that satisfies $c_{\ell-1} < \sum_{k\in K(u,v)} d^k \le c_\ell$.

To route all the traffic that belongs to $K(u,v)$, the capacity requirement for arc (u,v) is clearly c_ℓ and therefore we can fix the variables related to the redundant capacity levels to zero:

$$Y_{uv}^{\ell'} = 0, \ell' = \ell+1,\ldots,|L|. \tag{18}$$

Moreover, it is easy to verify that the commodities in $K(u,v)$ cannot use (u,v) simultaneously unless the capacity of (u,v) is at least c_ℓ. Hence we have the following valid inequality:

$$\sum_{k \in K(u,v)} \sum_{p \in P^k \cap P(u,v)} x_p - |K(u,v)| + 1 \leq Y^\ell_{uv}. \tag{19}$$

We note that (19) is also valid for any subset $K'(u,v) \subseteq K(u,v)$ with $\sum_{k \in K'(u,v)} d^k > c_{\ell-1}$, and that this inequality is particularly effective when the subset $K'(u,v)$ has the property that for any commodity $k' \in K'(u,v)$, $\sum_{k \in K'(u,v) \setminus \{k'\}} d^k \leq c_{\ell-1}$.

The inequalities below are special cases of (19), where the set $K(u,v)$ is restricted to contain one commodity k, and ℓ is the capacity level such that $c_{\ell-1} < d^k \leq c_\ell$.

$$\sum_{p \in P^k \cap P(u,v)} x_p \leq Y^\ell_{uv}, \forall k \in K, \forall (u,v) \in A. \tag{20}$$

By utilizing constraints (4), inequalities (20) can be alternatively stated in the wavelength assignment variables:

$$\sum_{h \in H} y^{kh}_{uv} \leq d^k Y^\ell_{uv}, \forall k \in K, \forall (u,v) \in A. \tag{21}$$

Note that the number of inequalities (20) is polynomial in the problem size, and hence it is convenient to explicitly add all these constraints to the model in advance.

Another class of valid inequalities can be derived for a set $K(u,v)$, for which we have $\sum_{k \in K(u,v)} d^k > |H|$. It is obvious that the commodities in $K(u,v)$ cannot use (u,v) simultaneously. We have hence the following inequality:

$$\sum_{k \in K(u,v)} \sum_{p \in P^k \cap P(u,v)} x_p \leq |K(u,v)| - 1. \tag{22}$$

As in the case of (20), inequalities (22) can also be derived for any subset $K'(u,v) \subset K(u,v)$.

A third set of inequalities concerns the wavelength assignment and the placement of OXCs (see also [5]). To obtain these inequalities, we consider an artificial, undirected graph $G^* = (V, E)$ which is derived from the original network $G = (N, A)$. We call G^* the *intersection graph of paths*, since a vertex in V corresponds to a path in G, and an edge in E corresponds to two paths in G that have at least one arc in common. Note that a path in G, or equivalently a node in G^*, is associated with a commodity. We let $k(p)$ denote the commodity of path p, i.e. p starts at s^k and ends at t^k. Due to the one-to-one relationship between the nodes in G^* and the paths in G, we will use indices p and q both to denote nodes in G^* and their corresponding paths in G. Consider a complete subgraph of G^*, such that the node set C in this subgraph corresponds to paths of different commodities in G, and the following two inequalities are satisfied:

$$\sum_{p \in C} d^{k(p)} > |H| \tag{23}$$

$$\sum_{p \in C \cap P(u,v)} d^{k(p)} \leq |H|, \forall (u,v) \in A. \tag{24}$$

If (24) does not hold for arc $(u, v) \in A$, the commodity demands can not be routed simultaneously on the paths sharing arc (u, v) since this is not allowed due to (22). It can be observed that if such a C exists, then wavelength switching is needed in the original network G if the corresponding paths are used. Note that without loss of generality, we may assume that $|C| \geq 3$. Indeed, inequalities (23) and (24) cannot hold simultaneously for $|C| = 2$. For cases of C with three or more elements, it is clear that without any wavelength switching, the paths represented by the nodes in C cannot be used simultaneously to carry flow. This implies that if these paths are to be used simultaneously, OXCs need to be installed at some of the nodes that are shared by these paths in the original network G. The valid inequality that follows from this observation is given below (for a more detailed treatment of the inequality, see [5]).

$$\sum_{p \in C} x_p - |C| + 1 \leq (\sum_{p \in C} \sum_{v \in N(C)} z_v^{k(p)})/(|C| - 2). \tag{25}$$

In the above inequality, v refers to a node in the original network G, and the set $N(C)$ is defined as $N(C) = \{v \in N : \exists p, q \in C \text{ with } v \text{ belonging to both paths } p \text{ and } q\}$.

A similar valid inequality can be derived for a cycle $O \in G^*$, if O contains an odd number of vertices, and the following inequality holds for any three consecutive vertices p, q and r in O.

$$d^{k(p)} + d^{k(q)} + d^{k(r)} > |H|. \tag{26}$$

In addition, we assume that

$$\sum_{p \in O \cap P(u,v)} d^{k(p)} \leq |H|, \forall (u, v) \in A. \tag{27}$$

We observe that wavelength switching is required if all paths corresponding to the nodes in O are to be used simultaneously. We have therefore the following valid inequality:

$$\sum_{p \in O} x_p - |O| + 1 \leq \sum_{p \in O} \sum_{v \in V(O)} z_v^{k(p)}. \tag{28}$$

Note that an inequality of (25) for which $|C| = 3$ is a special case of (28), for which the cycle O contains three elements.

3.1 Cutting planes

The solution approach that we use is based on a Branch and Bound procedure, in which the lower bound is initially computed by solving the LP-relaxation of the model given in Section 2.1. The LP-relaxation is then successively tightened by the addition of the valid inequalities discussed previously. In particular, all possible inequalities of classes (18) and (20) are generated and added to the model, since these two classes are relatively small in number. Valid inequalities of (19), (22), (25) and (28) are exponentially many, but usually only a subset of them is sufficient to significantly strengthen the lower bound. Therefore these constraints are generated by the cutting plane procedure that we briefly describe below.

The input to the cutting plane procedure is the optimal solution $\bar{X} = (\bar{x}, \bar{y}, \bar{z}, \bar{Y})$ of the LP-relaxation. An inequality of the general form $\alpha X \leq \alpha_0$ is said to be violated by $\bar{X}$ if $\alpha\bar{X} > \alpha_0$. The cutting plane procedure attempts to identify violated inequalities of previously discussed classes. To identify a violated ineuqality of a specific class, a so-called *separation problem* for that class is solved.

The separation problem for inequalities (19) can be formally stated as follows. Consider an arc (u, v) and a capacity level l, such that in the optimal LP-solution we have $0 < \bar{Y}^{\ell}_{uv} < 1$. In order to find a violated inequality, we select a subset of commodities, $K'(u, v) \subseteq K(u, v)$, such that $\sum_{k \in K'(u,v)} d^k > c_{\ell-1}$ and $\sum_{k \in K'(u,v)} \sum_{p \in P^k \cap P(u,v)} \bar{x}_p - |K'(u, v)| + 1 > \bar{Y}^{\ell}_{uv}$. The set $K'(u, v)$ can be found by solving the following binary problem:

$$\max \quad \sum_{k \in K(u,v)} \sum_{p \in P^k \cap P(u,v)} (\bar{x}_p - 1)\xi_k \tag{29}$$

$$\text{s.t.} \quad \sum_{k \in K(u,v)} d^k \xi_k \geq c_{\ell-1} + 1 \tag{30}$$

$$\xi_k \in \{0, 1\}, \forall k \in K(u, v) \tag{31}$$

where the variable ξ_k is equal to 1 if commodity k is included in $K'(u, v)$, and 0 otherwise.

The separation problem above is in fact a binary knapsack problem. This can be shown by the following variable transformation.

$$\xi_k = 1 - \beta_k, \forall k \in K(u, v). \tag{32}$$

In (32), β_k are the new variables which are clearly binary. By applying (32) to (29) and (30), together with some simple mathematical manipulations, the separation problem can be equivalently stated as:

$$\max \quad \sum_{k \in K(u,v)} \sum_{p \in P^k \cap P(u,v)} (1 - \bar{x}_p)\beta_k - \sum_{k \in K(u,v)} \sum_{p \in P^k \cap P(u,v)} (1 - \bar{x}_p) \tag{33}$$

$$\text{s.t.} \quad \sum_{k \in K(u,v)} d^k \beta_k \leq c_{\ell-1} + 1 + \sum_{k \in K(u,v)} d^k \tag{34}$$

$$\beta_k \in \{0, 1\}, \forall k \in K(u, v). \tag{35}$$

Observe that the coefficients $(1 - \bar{x}_p)$ are non-negative, and the second term in (33) is a constant. Therefore (33)-(35) form a standard binary knapsack problem, in which the number of variables is less or equal to the number of commodities. Although this problem is $\mathcal{NP}$-hard, it can be solved efficiently for the network sizes that we consider (see e.g. [12]). Note that a violated inequality of class (19) is found if the optimal objective function value is greater than $\bar{Y}^{\ell}_{uv} - 1$.

A similar separation scheme can be used to identify violated inequalities of class (22). In particular, the following binary problem, which can be transformed into a standard binary knapsack problem, must be solved to determine whether any inequality of (22) is violated for arc (u, v):

$$\max \quad \sum_{k \in K(u,v)} \sum_{p \in P^k \cap P(u,v)} (\bar{x}_p - 1)\eta_k \tag{36}$$

$$\text{s.t.} \quad \sum_{k \in K(u,v)} d^k \eta_k \geq |H| + 1 \tag{37}$$

$$\eta_k \in \{0, 1\}, \forall k \in K(u, v).$$

A violated inequality of (22) is found if the above problem yields an objective function value that is greater than -1.

Violated inequalities of classes (25) and (28) can be separated by utilizing the intersection graph of paths, G^*, in which a node p (which is a path in G) is associated with a weight p^*. The separation problem for (25) concerns finding maximum-weighted complete subgraphs of G^*, such that the nodes in the subgraphs belong to different commodities in the original graph, and (23) is satisfied. Instead of examining complete subgraphs, the corresponding graph structure to be examined in the separation problem for (28) is odd cycles. These two separation problems are difficult to solve in general. However, the size of the graph G^* for practical instances of optical network design is usually quite limited. The separation of (25) can often be done exactly by identifying all the maximum cliques by the algorithm provided in [10] or by enumeration. A simple enumeration of odd cycles of small dimensions in G^* can be used to identify potential inequalities of (28).

4 Computational results

To investigate the efficiency of the proposed integer model and the valid inequalities described in Section 3, computational experiments have been conducted for a number of scenarios of metropolitan and wide area networks. In these experiments we use a standard modeling language (AMPL) and a commercial MIP solver (CPLEX 6.6).

The first three scenarios are derived from a metropolitan network that has 11 nodes, with node distances of up to 16 miles. In the first scenario, the numbers of arcs, commodities and paths are 42, 22, and 136 respectively. Demands vary from 1 to a maximum of 5 wavelength channels. No OXC is used in the optimal solution and the routing follows more or less the shortest paths. In order to investigate whether OXCs are necessary for higher traffic demands in this network, scenarios 2 and 3 have been generated, where the numbers of arcs and commodities are 43 and 25, and the number of paths has been increased to 142. The maximum demand is 13 wavelength channels. In scenario 2, the OXC cost is very high, which makes it more economical to install additional DWDM capacities rather than OXC facilities, while in scenario 3 the OXC facility is almost free of charge.

The second network used in the computational experiments is the wide area NSF-network with 14 nodes and 54 arcs. The number of commodities is 35, and the demands range from 1 to 17. The total number of paths generated is 110. Two scenarios with different OXC costs are considered for this network. In scenario 4, the OXC is very expensive, while in scenario 5 the OXC cost is negligible.

Six DWDM technologies with capacities 1, 2, 4, 8, 16, and 32 wavelengths are available for each arc in all the scenarios. For the first capacity step (i.e. for installing a fiber with capacity 1), the cost is 10000 times the length in miles. To increase the capacity from 1 to 2, 2 to 4, 4 to 8, 8 to 16 and finally 16 to 32, the additional costs are 2000, 3000, 10000, 20000 and 60000 respectively. Despite the relatively small sizes of the networks, the resulting models have more than 7000 binary variables and 14000 constraints for the first three scenarios, and 10000 binary variables and 17000 constraints for scenarios 4 and 5 before any valid inequality is added.

For each scenario, the computational experiments are conducted in the following steps.

1. We solve the LP-relaxation of the model defined by (14)–(17). This is the original linear integer model without any additional valid inequalities. Let v_{org}^{LP} denote the objective function value.

2. An attempt is then made to solve the linear integer program to optimality using the standard MIP solver that starts from the optimal solution of the LP-relaxation obtained in Step 1. The time limit is set to 10 hours for this step. Let v_{org}^{IP} denote the best integer solution found by the MIP solver within the time limit.

3. We then add the valid inequalities of types (18) and (20), which result in a stronger problem formulation. Let the corresponding LP-value be v_{str}^{LP}.

4. In this step, a cutting plane procedure is used to generate violated valid inequalities of (19), (22), (25) and (26). We separate valid inequalities of (25) and (26) manually since they are strongly related to the network structure. Valid inequalities of (19) and (22) are generated by solving the corresponding separation problems described in Section 3.1. The objective function value obtained after adding valid inequalities of (19), (22), (25) and (26) is denoted by v_{cut}^{LP}.

5. The purpose of this step is to find a good upper bound (i.e. a feasible integer solution), which together with the lower bounds found in the previous steps, guarantees the quality of the solution and enables a faster convergence of the algorithm in Step 6. We generate integer solutions by rounding off the LP-solutions in Steps 1 and 3. For each commodity, the path variable that attains the largest value in the LP-solutions is selected and set to one, if this, together with previous selections of the path variables, does not imply a flow greater than $|H|$ on any arc. Since the routing solutions can be determined for most commodities in this way, the remaining problem can be solved efficiently by the MIP solver. It was found that the stronger LP in Step 2 always yields a better upper bound, which is denoted by v_{heu}^{IP}.

6. Starting from the LP-solution found in Step 4 and the integer solution found in Step 5, the MIP solver is applied for solving the linear integer problem to optimality. We use v_{fin}^{IP} to denote the optimal objective function value of this step.

We summarize the computational results in Table 1. The computation time required for each step is displayed by the time entries. For Steps 1, 3, 4 and 5, the gap is measured with respect to v^{IP}_{fin}, which is actually optimal for all the scenarios. In Step 2, the gap is the remaining gap reported by CPLEX after running for 10 hours. For Steps 2 and 6, the numbers of Branch and Bound nodes are displayed. The table also shows the total number of valid inequalities generated in Step 4 for each scenario. Most of these cuts are of type (19). For scenarios 2 and 3, one cut of (25) is found, and for scenario 3 one cut of (22) is generated. For scenarios 4 and 5, one cut of class (25) is added, and two cuts of (22) are generated. All the computations have been conducted on a SUN UltraSparc station with a 400 MHz CPU and 2 GB physical memory. In addition, default parameter settings in the MIP solver have been used.

	Metropolitan network			NSF network	
	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5
Step 1: v^{LP}_{org}	207850	513787.5	513787.5	945312.5	945312.5
Time	1.2s	1.6s	1.5s	4.7s	4.6s
Gap	72.81%	51.86%	49.50%	41.90%	40.43%
Step 2: v^{IP}_{org}	787500	1410200	1278801	4400000	1722001
Time	10h	10h	10h	10h	10h
Nodes	6382	3234	3431	6614	7068
Gap	6.32%	31.79%	24.63%	66.74%	11.14%
Step 3: v^{LP}_{str}	733000	945083.33	945084.33	1400750	1400750
Time	11s	17s	16s	7s	7.3s
Gap	4.12%	11.46%	7.12%	13.91%	11.74%
Step 4: v^{LP}_{cut}	752827.31	1057581.67	1004532.67	1571416.98	1531390.50
Time	15m11s	12m37s	12m5s	29m59s	30m9s
Cuts	57	43	50	56	62
Gap	1.53%	0.93%	1.27%	3.42%	3.50%
Step 5: v^{IP}_{heu}	764500	1072500	1017501	1627000	1627000
Time	57s	99s	150s	53s	53s
Gap	0	0.47%	0	0	2.52%
Step 6: v^{IP}_{fin}	764500	1067500	1017501	1627000	1587001
Time	20m	56m40s	1h41m20s	6m20s	38m20s
Nodes	67	134	180	14	107

Table 1: A summary of the computational results.

Although the bounds provided by the original LP-relaxations are easy to compute, they are very weak for all the scenarios. The integrality gap is between 40% and 70%. When no valid inequalities of the proposed classes are added, CPLEX is not able to close the gap for any scenario after 10 hours. The gap between the lower bound and upper bound found in Step 2 is still quite large and lies between 6% and 66%. Furthermore, the best integer solution found by CPLEX in Step 2 is constantly worse than the one found by the heuristic in Step 5. The addition of valid inequalities of (18) and (20) has a great impact on the lower bound, for instance the gap decreases from 72% to about 4% for scenario 1. Comparing with Step 1, the gap is reduced by 79.12% on average. Even though the cutting plane procedure in Step 4 requires a relatively large amount of time (the code however is not optimized), the solution improvement is significant and the gap can be reduced to less than 3.5%, or even less than 1% in some cases. A comparison with Step 1 shows that the gap is closed by 95.34% on average. We observe that the heuristic upper bound computed by starting from the stronger LP-relaxation, v_{heu}^{IP}, is optimal for three of the scenarios and near-optimal for the other two scenarios. Equipped with all the valid inequalities in Steps 3 and 4, and the heuristic upper bound found in Step 5, CPLEX is able to close the gap, and find or verify optimum for all the scenarios within a reasonable computation time. As expected, no OXC is needed in scenario 1. The optimal solutions of scenarios 2 and 4 do not contain any OXC due to its high cost, while one OXC is used in scenarios 3 and 5. It is observed that the incremental cost of not using any OXC equipment is slightly less than the average cost of establishing a link, and corresponds to a few percent of the total design cost.

The progress of the cutting plane procedure is shown in Figure 1, where the integrality gap values of the LP-solutions are plotted for all the scenarios. The number of cutting plane iterations lies between 7 and 14, and most cuts and gap reductions are obtained in the first five iterations.

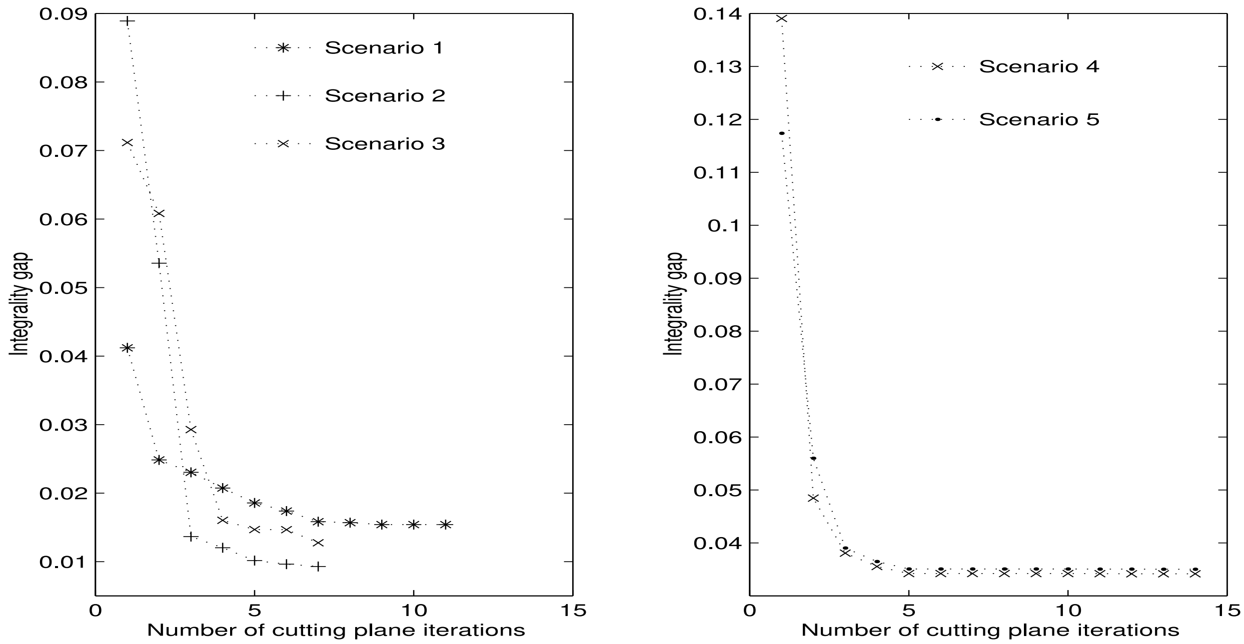

Figure 1: The progress of the cutting plane procedure.

5 Conclusions and future work

We have presented a new mathematical programming model for design, routing, and wavelength assignment in optical networks. Several classes of valid inequalities are introduced and the corresponding separation procedures are discussed. The solution approach combines a cutting plane procedure and a commercial MIP solver. Computational experiments are conducted for a number of scenarios of metropolitan and wide area networks. The computational results show that the proposed linear integer model cannot be solved directly without adding any valid inequalities. However, the LP-relaxation can be significantly improved by incorporating the cutting plane procedure which yields a much tighter formulation that can be solved to optimality by the MIP solver within a reasonable amount of computation time. It is therefore practical to apply mathematical programming techniques to this type of optical network planning problem. Another observation is that unless optical switches are very cost-attractive compared to installing additional link capacity, they are not often used in the optimal network solution.

At the moment, the set of available paths in the mathematical model is generated in advance. In the future, we plan to improve the solution approach for the LP-relaxation by applying a combined column and cut generation technique in which the path variables are generated dynamically. The computational results indicate that for large scale instances it is probably necessary to develop advanced heuristic solution procedures. Another interesting part of the future work will be to develop and solve more complex mathematical models that incorporate path protection and restoration.

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