

PERIODIC SERVICES' SCHEDULING IN RAILWAY FREIGHT TRANSPORTATION¹

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Abstract: In this paper we address a particular periodic scheduling problem arising when designing timetables in the railway freight transportation setting. Given a railway network, a set of commodities to be transported in the network, and a set of train services with their frequencies, the problem consists in determining the departure times for each train and the route for each commodity to minimize the total delay time incurred by the commodities at the stations. Two alternative models are proposed, which are used to obtain both lower bounds and heuristic solutions.

Keywords: Freight transportation, periodic scheduling, routing, timetable synchronization.

1. Introduction

In the medium range planning of the activities in a Railway Freight Transportation System, one of the most relevant problems is to define the service level, that is the train itineraries, called *services*, and their frequencies (number of trips in the time horizon). This is also called *tactical planning*. Once the service level has been established, at operational level (*short range planning*) the problem is to determine the departure times of the trains from each station of their itinerary, and to decide the routing of the shipped goods so that the global waiting time at the stations is minimized. This paper will focus on this second problem, which can be seen as a combination of a routing and a periodic scheduling problem. In particular we will be interested in minimizing the total delay cost due to cars waiting for connections in intermediate stations. In Section 2, we rapidly review the tactical planning problem in order to establish the notation we will use; we also briefly review some of the literature on this field; then, in Section 3, we present the operational planning problem, for which two models are proposed. For the first model, a relaxation which allows us to obtain lower bounds is described in Section 4. In Section 5 heuristic algorithms based on the two models are presented. A preliminary computational comparison on real data is discussed in Section 6.

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2. Tactical planning

Here we briefly review the medium range planning problem and introduce the basic notation used throughout the paper. A Railway Freight Transportation System is defined by a physical network, $G = (N, A)$, where the nodes correspond to stations and the arcs to railway tracks, and by a set of Origin/Destination (O/D) pairs; for each O/D pair the demand of transportation is known. In particular the demand is specified by the quantity of goods to be shipped from the origin to the destination and by the type of goods. Indeed different type of goods can induce different transportation costs (e.g. perishables, or goods with high priority).

In the following we consider a *commodity* as defined by an Origin/Destination pair and by one type of good; so the number of commodities is bounded by the number of O/D pairs times the number of distinct goods. The demand of transportation for each commodity is given in *carloads*, i.e. in number of cars, to be transported within the planning horizon. Since, most often, the train schedule is periodic with period one week, the week will be taken as the planning horizon. Of course any other planning horizon may be used instead.

We shall assume that the empty car redistribution has already been included in the demand, that is the empty car will be considered as a particular commodity to be transported. The problem of determining the fleet size and the redistribution of empty cars is an interesting and challenging problem, which has been already widely studied (Beaujon and Turnquist, 1991; Hagani 1989).

The stations are of different kinds depending on the type of facilities available. The most expensive operation which may occur at a station is the so called *classification*: in this operation, using different parallel tracks, cars are sorted in such a way that, at the end of the operation, each track contains cars of the same type, i.e. cars with a common (possibly intermediate) destination. Each line of cars of the same type produced by the classification operation will be called a *block*. The classification operation requires a large yard with many tracks, so it can be performed only at some stations. Another operation (much less space and time consuming) is the operation of attaching to or detaching from a train a block of cars. Also these problems have been studied extensively in the literature (see for example Crainic, Ferland and Rousseau, 1986; Marin and Salmeron, 1993). In our models we assume a fixed amount of time for the classification operation, depending on the station.

A *train service* (simply, a *service*) is a trip performed by a single train, which may or may not stop at intermediate stations between the Origin and the Destination:

The objective of the tactical planning is to determine, within the planning horizon, the train services to be run and their frequencies (i.e. number of runs within the time horizon).

A typical approach to the tactical planning described above is the one proposed by Crainic and Rousseau (1984). In that paper the authors propose a heuristic based on the computation of minimum cost multicommodity flows on a suitable and usually very large network.

A similar problem is addressed also in (Keaton, 1992), where the tactical planning of a particular railway system is efficiently solved by means of a heuristic algorithm based on a Lagrangean relaxation.

The tactical planning in rail freight networks has been extensively studied by Marin and Salmeron (1996 a, b, c).

Note that, in the tactical planning phase, the railway managers are not interested in the exact timetable of each train: indeed they are rather interested in the *itineraries* and in the *services' frequencies*, that is the number of service instances that have to be activated during the planning period.

3. The scheduling problem

Here, we consider the problem of determining the actual schedule for the services which have been chosen by solving the tactical planning (see Section 2). Such services will be called *active services*; they are defined as sequences of *legs*, a leg being the portion of service between two consecutive stations. Remembering that the schedule should be periodic with period U (usually one week), the problem is to assign a departure time within such period to each instance of each active service; for each service we have as many instances as is its frequency value. The main constraints to be considered in solving the scheduling problem are the following:

- the demand of transportation for each commodity must be satisfied,
- at any point in time, the stations and track capacities cannot be exceeded,
- each service instance cannot carry more than a given number of cars,
- the interval between the departure times of consecutive instances of the same service is bounded from below.

In the solution of the scheduling problem the objective is to minimize the global time spent by cars in the stations. In fact, while we do not have control on the traveling times, which depend on the available technology, a clever management of the operations performed at the stations and a good timetabling can significantly curtail the commodities' travel time.

We propose two models to describe the problem: the first one, based on a discretization of the time horizon, is a multicommodity flow model on a quite large network, while the second, in which the station capacity constraints (quite loose in most practical cases) are relaxed, is of a much smaller (and hence tractable) size, but has a quadratic objective function.

It should be noted that, in our models, we are not explicitly considering the track capacity constraints which may have a practical relevance. Indeed the solution yielded by the proposed models might present some “conflicts”, that is more trains present at the same time in the same track section. Solutions violating these constraints can be suitably adjusted by applying a *spacing phase* (Chen and Harker 1990; Kraay, Harker and Chen 1991). This phase is particularly important when single track networks are considered or when the railway is shared by freight transportation and passenger transportation. In all the experiments carried out in this paper, we have never had to adjust the solution to eliminate conflicts. This problem is also considered in (Caprara et. al. 1998) and in (Brunetta, Colorni, Laniado 1999).

3.1 A network flow model

To make the problem more tractable, we assume the time to be discretized by partitioning the period into T intervals of equal length with starting points $0, 1, \dots, T-1$. The intervals' length will be taken as the time unit. As an example, if the period U corresponds to one week, and if the intervals' length is of one hour, then time 0 corresponds to 0:00 a.m. of Monday and time T corresponds to 12:00 p.m. of Sunday.

The input data for this model are: the set of all possible commodities W , the demand d_w in number of cars during the period, for each commodity $w \in W$, the set of active services S_a and, for each service $s \in S_a$, the frequency y_s (i.e. the number of copies of service s to be activated during the period). For each $w \in W$, we will denote by $n_o(w)$ the station at which commodity w is originated and by $n_d(w)$ the destination station of the same commodity. A leg l shall be denoted by the triple (n', n'', δ_l) , where n' and n'' are the starting and ending stations of the leg and δ_l is its duration which includes also the time needed to load the cars at the beginning of the trip (including classification time) and the time needed to unload it at the end of the trip.

We shall call *service instance* a pair (s, t) , where s is an element of S_a and $t \in \{0, 1, \dots, T-1\}$ is the departure time from the origin of s . The set of all possible service instances is $\Sigma = S_a \times \{0, 1, \dots, T-1\}$.

The model we are going to propose is based on the definition of the *time expanded graph* $G^* = (N^*, A^*)$, where:

$$N^* = \{(n, t): n \text{ is a station and } t \in \{0, 1, \dots, T-1\}\} \\ \cup \{n_o(w): w \in W\} \cup \{n_d(w): w \in W\},$$

and

$$A^* = A_1 \cup A_2 \cup A_3 \cup A_4;$$

with

$$\begin{aligned}
 A_1 &= \{((n', t'), (n'', t'')): \exists s \in S_a \text{ and } \exists l \in s \text{ s.t. } l = (n', n'', (t'' - t') \bmod T)\}, \\
 A_2 &= \{((n, t), (n, (t+1) \bmod T)), t = 0, 1, \dots, T-1\}, \\
 A_3 &= \{(n_o(w), (n_o(w), t)): w \in W, t \in \{0, 1, \dots, T-1\}\}, \\
 A_4 &= \{(n_d(w), t), n_d(w)): w \in W, t \in \{0, 1, \dots, T-1\}\}.
 \end{aligned}$$

The arcs in A_1 correspond to legs, i.e. to possible physical movements of commodities between two stations, those in A_2 correspond to waiting times, while those in A_3 and A_4 are related to loading and unloading operations, respectively.

Note that a service instance (s, t) defines in G^* in a unique way a path going from the node (n, t) corresponding to the initial station at time t , to the node (n', t') corresponding to the terminal station, and $(t' - t) \bmod T$ is the service duration. We will call $\rho(s, t)$ such path.

Before writing the constraints of the model of the scheduling problem, we need some further notation:

- h_{rw} : variable expressing the flow (number of cars) of commodity w on route r ;
- d_w : demand for commodity w ;
- z_{st} : 0/1 variable (=1 iff service instance (s, t) is chosen);
- E_w : set of routes that can be used by commodity w ; a route feasible for commodity w is a path in graph G^* starting in $n_o(w)$ and ending in $n_d(w)$
- \bar{a}_n : capacity (number of cars) of station n for a single time interval $[t, t+1]$;
- R_{nt} : set of routes using station n at time t ;
- R_l : set of routes using leg l ;
- R_e : set of routes using arc $e \in A^*$;
- y_s : frequency of service s (number of trains per period);
- B_s : maximum number of cars per train in service s ;
- $S_k(t)$: set of all the service instances which pass through track k at time t ;
- T_s : minimum time distance between two consecutive departure of service s .

The model is:

$$\sum_{r \in E_w} h_{rw} = d_w, \quad \forall w \in W, \quad \text{[demand of transportation]} \quad (3.1)$$

$$\sum_{w \in W} \sum_{r \in E_w \cap R_{nt}} h_{rw} \leq \bar{a}_n, \quad \forall (n, t) \quad \text{[station capacity]} \quad (3.2)$$

$$\sum_{w \in W} \sum_{r \in E_w \cap R_e} h_{rw} - B_s z_{st} \leq 0 \quad \forall e \in \rho(s, t), \forall (s, t) \in \Sigma \quad \text{[service capacity]} \quad (3.3)$$

$$\sum_{t=0}^{T-1} z_{st} = y_s \quad \forall s \in S_a \quad \text{[frequency]} \quad (3.4)$$

$$\sum_{i=1, \dots, T_s} z_{s, t+i \bmod T} \leq 1, \quad \forall (s, t) \in \Sigma \quad \text{[departure times]} \quad (3.5)$$

$$\begin{aligned}
 z_{st} &\in \{0, 1\}, & \forall (s, t) \in \Sigma \\
 h_{rw} &\geq 0, & \forall w \in W, \forall r \in E_w.
 \end{aligned}$$

Constraints (3.5) assure a proper spacing between successive departures of instances of the same service. Note that the *modulo* operation is necessary to take into account the fact that the scheduling is periodic.

The objective function to be minimized is the total cars' delay cost. Since the flow on the legs' arcs corresponds to movements on tracks and to operations at the stations, which must be performed and whose duration is independent from the schedule chosen, the only flow which gives a contribution to our objective function is the flow on arcs in A_2 ; let us denote by c_w the cost per car and per unit of delay of commodity w . The objective function is:

$$\text{Min} \sum_{w \in W} \sum_{(i,j) \in A_2} \sum_{r \in E_w: (i,j) \in r} c_w h_{rw} \quad (3.6)$$

The resulting problem is a minimum cost multicommodity flow problem with side constraints which include some *network design* features. We have chosen a *path formulation* of the multicommodity portion of the model; obviously, an *arc formulation* could have been used as well.

3.2 A quadratic model

A critical point of the model described in the previous section is the size which, for real applications, can be quite large. Here we investigate an alternative model which leads to the solution of problems of relatively small size. The model that we are going to propose in this section is equivalent to the one presented above, except for the fact that we relax the station capacity constraints. In fact, on one side, the introduction of these capacities would make the problem much more difficult to deal with, and on the other side, once a solution has been obtained it is quite simple to adjust it, by properly shifting the departure times of the conflicting service instances, to make it feasible, if needed.

Let us call *active services' graph* a directed graph $G_A = (N_A, N_A)$ where the nodes in N_A correspond either to Origins/Destinations or to stations of the physical network which are used by some active services, and there is an arc between two nodes n' and n'' if there exists an active service containing a leg (n', n'', δ) , for some δ , or n' is an origin and n'' is the corresponding loading station, or n'' is a destination and n' is the corresponding unloading station.

For each $s \in S_{ar}$ let us denote by $I_s = \{s_1, s_2, \dots, s_{y_s}\}$ the set of its instances, and by I the set of the instances of all the active services. Note that, with this notation, an instance has not a time attached as in the previous model but it has a position: instance s_i is the i^{th} instance of service s in the planning period, and we want to assign to each instance s_i , $i=1, \dots, y_s$, a starting time π_i such that $\pi_1 < \pi_2 < \dots < \pi_{y_s}$. As before the objective is to minimize the global delay cost, i.e. the

time spent in transferring commodities from one train to another, multiplied by the number of carloads and the cost coefficient c_w . Note that variables π_i are continuous, hence, unlike the previous case, train departure times can assume real values in the interval $[0, \dots, T)$. Given a route r and two service instances i and j which are used consecutively by route r , $k(i,j,r) \in N_A$ defines the common station where commodities are unloaded from i and loaded in j . Without loss of generality, we can assume that for each pair of service instances i and j which are used consecutively on route r , the exchange station $k(i,j,r)$ is unique. In addition, let $C(r)$ denote the set of pairs of service instances used consecutively in route r , and let ε_{ik} be the trip duration (including loading and unloading times) from the first station of instance service i to station k . The model is defined as follows:

$$\text{Min} \quad \sum_{w \in W} \sum_{r \in E_w} \sum_{i,j \in C(r)} (\pi_j - \pi_i + Tx_{ijk(i,j,r)} - \varepsilon_{ik(i,j,r)} + \varepsilon_{jk(i,j,r)}) c_w h_{rw} \quad (3.7)$$

$$\sum_{r \in E_w} h_{rw} = d_w, \quad \forall w \in W, \quad (3.8)$$

$$\sum_{w \in W} \sum_{r \in E_w \cap R_l} h_{rw} \leq B_l, \quad \forall l \in I, \forall i \in I, \quad (3.9)$$

$$h_{rw} > 0 \Rightarrow \pi_j - \pi_i + Tx_{ijk(i,j,r)} \geq (\varepsilon_{ik(i,j,r)} - \varepsilon_{jk(i,j,r)}), \quad \forall r \in E_w, \forall i,j \in C(r), \quad (3.10)$$

$$\pi_j - \pi_{j-1} \geq T_s, \quad j=2, \dots, y_s, \quad \forall s \in S_a \quad (3.11)$$

$$\pi_1 - \pi_{y_s} + T \geq T_s, \quad \forall s \in S_a, \quad (3.12)$$

$$\begin{aligned} x_{ijk(i,j,r)} &\in \{0,1\}, & \forall w \in W, \forall r \in E_w, \forall i,j \in C(r), \\ h_{rw} &\geq 0, & \forall w \in W, \forall r \in E_w. \end{aligned}$$

In the objective function $(\pi_j + \varepsilon_{jk(i,j,r)}) - (\pi_i + \varepsilon_{ik(i,j,r)})$ gives the difference between the arrival time of instance i and instance j at station $k(i,j,r)$. Variable $x_{ijk(i,j,r)}$ will be set to one if i arrives at the connection station $k(i,j,r)$ after j has already left; in this case the involved commodities will wait until service j passes through the station again in the next period. The usual constraints on the transportation demand are specified by (3.8) and service capacity constraints are given by (3.9), while departure times spacing constraints are expressed by (3.11) and (3.12). Logic constraints (3.10) state that if route r is used by some commodities then service instances $i, j \in C(r)$ must be in connection. A suitable linearization of these constraints may be obtained easily by introducing a set of 0-1 variables.

4. A lower bound based on Lagrangean relaxation

In the flow model proposed in Section 3.1, the only constraints binding flow variables h to the 0-1 variables z are constraints (3.3). Applying Lagrangean relaxation to those constraints, we obtain the following relaxed problem, where $\lambda \geq 0$:

$$\varphi(\lambda) = \text{Min} \sum_{w \in W} \sum_{(i,j) \in A_2} \sum_{r \in E_w: (i,j) \in r} c_w h_{rw} + \sum_{(s,t) \in \Sigma} \sum_{e \in \rho(s,t)} \lambda_e \left(\sum_{w \in W} \sum_{r \in E_w \cap R_e} h_{rw} - B_s z_{st} \right) \quad (4.1)$$

$$\sum_{r \in E_w} h_{rw} = d_w, \quad \forall w \in W, \quad (4.2)$$

$$\sum_{w \in W} \sum_{r \in E_w \cap R_{nt}} h_{rw} \leq \bar{a}_n, \quad \forall (n,t), \quad (4.3)$$

$$\sum_{t=0}^{T-1} z_{st} = y_s, \quad \forall s \in S_a, \quad (4.4)$$

$$i=1, \dots, T_s \sum_{\tau=(t+i) \bmod T} z_{s\tau} \leq 1, \quad \forall (s,t) \in \Sigma, \quad (4.5)$$

$$z_{st} \in \{0,1\}, \quad \forall (s,t) \in \Sigma,$$

$$h_{rw} \geq 0, \quad \forall w \in W, \forall r \in E_w.$$

The Lagrangean Dual is then $\text{Max} \{ \varphi(\lambda) : \lambda \geq 0 \}$.

For any given λ , $\varphi(\lambda)$ can be computed by solving two independent subproblems. The first subproblem involving variables h is a standard multicommodity flow which can be solved rather efficiently (Ahuja, Magnanti & Orlin, 1993). The minimum cost multicommodity flow problem is:

$$\text{Min} \sum_{w \in W} \sum_{(i,j) \in A_2} \sum_{r \in E_w: (i,j) \in r} c_w h_{rw} + \sum_{(s,t) \in \Sigma} \sum_{e \in \rho(s,t)} \lambda_e \sum_{w \in W} \sum_{r \in E_w \cap R_e} h_{rw}$$

$$\sum_{r \in E_w} h_{rw} = d_w, \quad \forall w \in W,$$

$$\sum_{w \in W} \sum_{r \in E_w \cap R_{nt}} h_{rw} \leq \bar{a}_n, \quad \forall (n,t),$$

$$h_{rw} \geq 0, \quad \forall w \in W, \forall r \in E_w.$$

The second subproblem is:

$$\text{Min} \sum_{(s,t) \in \Sigma} \sum_{e \in \rho(s,t)} -\lambda_e B_s z_{st}$$

$$\sum_{t=0}^{T-1} z_{st} = y_s, \quad \forall s \in S_a, \quad (4.6)$$

$$i=1, \dots, T_s \sum_{\tau=(t+i) \bmod T} z_{s\tau} \leq 1, \quad \forall (s,t) \in \Sigma, \quad (4.7)$$

$$z_{st} \in \{0,1\}, \quad \forall (s,t) \in \Sigma.$$

This last problem deserves some attention due its particular structure. In fact, it decomposes into a set of separable subproblems, one for each service $s \in S$:

$$\begin{aligned} \text{Min} \quad & \sum_{t=0}^{T-1} \sum_{e \in \rho(s,t)} -\lambda_a B_s z_{st} \\ & \sum_{t=0}^{T-1} z_{st} = y_s \end{aligned} \tag{4.8}$$

$$\begin{aligned} & \sum_{t=1, \dots, T_s} z_{s\tau} \leq 1, \quad i = 0, \dots, T-1 \\ & z_{st} \in \{0,1\}, \quad \forall (s,t) \in \Sigma. \end{aligned} \tag{4.9}$$

In practice we have to determine the departure time of y_s trains properly spaced and such that the sum of the costs associated to each departure time is minimized. The optimal solution can be found in pseudo-polynomial time by solving T_s constrained shortest path algorithms on the graphs $G_{sj} = (N_{sj}, A_{sj})$, $j=0, \dots, T_s-1$, defined as follows:

$$\begin{aligned} N_{sj} &= \{o, d\} \cup \{j, j+1, \dots, T-1\}, \\ A_{sj} &= \{(o,t): t=j, \dots, T-1\} \cup \{(t,d): t \leq T-T_s+j\} \cup \{(t,t'): t'-t \geq T_s\}. \end{aligned}$$

The weight associated with each node $t \in N_{sj}$ is

$$w_t = \begin{cases} 0 & \text{if } t=o, d \\ \sum_{e \in \rho(s,t)} -\lambda_e B_s & \text{otherwise.} \end{cases}$$

On graph G_{sj} , each path from o to d of exactly y_s+1 arcs, corresponds to a properly spaced departure times assignment to y_s trains such that no train will leave before time j . The path of exactly y_s+1 arcs which minimizes the sum of the node weights defines the optimal departure time assignment of trains leaving not before time j . This path can be found in polynomial time by means of the following procedure (Pratesi, 1995):

```

procedure Fixed_length_path ( $G_{sj}$ );
begin
  { data structure initialization}
  pred( $d$ ):= $\emptyset$ ; label( $d$ ):= $\infty$ ;
  for  $i=j, \dots, T-1$  do
    begin
      pred( $i,1$ ):= $o$ ; label( $i,1$ ):= $w_i$ ;
      for  $k=2, \dots, y_s$  do
        begin
          pred( $i,k$ ):= $\emptyset$ ; label( $i,k$ ):= $\infty$ ;
        end
      end
    {fixed length path computation}
    for  $i=j, \dots, T-1$  do
      begin
        for  $k=1, \dots, y_s-1$  do
          begin
            if pred( $i,k$ ) $\neq\emptyset$  then
              for  $h=i+T_s, \dots, T-1$  do
                if label( $i,k$ )+ $w_h <$  label( $h,k+1$ ) then
                  begin
                    pred( $h,k+1$ ):= $i$ ; label( $h,k+1$ ):=label( $i,k$ )+ $w_h$ ;
                  end
                end
              end
            if pred( $i,y_s$ ) $\neq\emptyset$  and ( $i \leq T-T_s+j$ ) and label( $i,y_s$ ) $<$ label( $d$ ) then
              begin
                pred( $d$ ):= $i$ ; label( $d$ ):=label( $i,y_s$ );
              end
            end
          end
        end
      end
    end.

```

Paths are described by a predecessor function $\text{pred}(i,k)$ that gives the predecessor of node i in the path from o , which uses exactly k arcs. The total weight of the path is given by $\text{label}(i,k)$. At the end of the procedure, $\text{pred}(d)$ contains the minimum weight path whose total weight is given by $\text{label}(d)$.

Note that graphs G_{sj} are acyclic, thus the computational complexity of procedure **Fixed_length_path** (G_{sj}) is $O(|A_{sj}| y_s)$. To solve the whole problem it suffices to call the procedure **Fixed_length_path** (G_{sj}) for $j=0, \dots, T_s-1$; the minimum weight path defines the minimum cost departure time assignment for service s . Hence the overall complexity is $O(T_s |A_{sj}| y_s)$.

The optimal solution of the Lagrangean Dual can be computed by efficient algorithms, see for example (Carraraesi, Frangioni and Nonato 1996). This optimal solution gives a lower bound for the total delay of the transportation system.

5. Heuristic solutions to the scheduling problem

5.1 A heuristic for the flow model

Here we propose a heuristic approach to find a feasible solution for the scheduling problem starting from the optimal solution of the Lagrangean relaxation. Such a solution can be

constructed heuristically by means of network flow algorithms along the lines proposed in (Bertossi, Carraresi and Gallo, 1987) for vehicle scheduling problems.

Let λ^* be the optimal Lagrangean multipliers vector, and (h^*, z^*) be the solution found evaluating $\varphi(\lambda^*)$. If (h^*, z^*) satisfies the service capacity constraints (3.3), then we have found the optimal solution. On the contrary, if some constraints are violated we have:

$$\sum_{w \in W} \sum_{r \in E_w \cap R_e} h_{rw}^* - B_s z_{st}^* > 0 \quad (5.1)$$

for some $e \in \rho(s, t)$, and $(s, t) \in \Sigma$; we can try to modify flow variables h^* in order to obtain a feasible flow with respect to the given z^* . This can be done by repeating the following steps, until a feasible solution is obtained:

- 1) select an arc $e=(i, j) \in A_1$ such that (5.1) holds;
- 2) while the constraint is violated repeat:
 - 2.1) select one route r among all the routes with $h_{rw}^* > 0$ that use e ,
 - 2.2) decrease of $\Delta = \min\{h_{rw}^*, B_s z_{st}^* - \sum_{w \in W} \sum_{r \in E_w \cap R_e} h_{rw}^*\}$, the flow on the path from i to the destination of commodity w ,
 - 2.3) redistribute the Δ units of flow on the shortest paths from node i to the destination having positive residual capacities, with respect to the current flow.

Given a solution h , let f_e denote the amount of flow on arc e . The paths of step 2.3, contain only arcs with positive residual capacity; that is one arc $e=(i, j)$ can be used from i to j if the flow on the arc, f_e , does not exceed the capacity, or from j to i if the flow is strictly positive.

In other words, the residual capacities of the arcs with respect to h are given by f_e , if used in inverse direction, and by:

$$r_e = \begin{cases} \bar{a}_n - f_e & \text{if } e \in A_2, \\ B_{st} z_{st}^* - f_e & \text{if } e=(i, j) \in \rho(s, t), \\ \infty & \text{for all the other arcs } e \in A^*, \end{cases}$$

in the other cases.

The costs that are used for the shortest path computations, are given by the reduced costs with respect to the optimal Lagrangean multipliers λ^* . The optimality of multipliers assures that all the costs be non negative.

Obviously, the final solution provided by the heuristic is strongly affected by the order in which violated constrains are considered, or by which commodity is selected to be re-routed to the destination. In our implementation we did not apply any particular strategy.

5.2 A heuristic for the quadratic model

A heuristic approach for the quadratic model can be derived from the following observations:

(i) when departure times are fixed the problem becomes a standard multicommodity flow

with linear objective function, while (ii) when the flow variables are fixed, the problem is a relatively small size mixed integer linear problem.

The first subproblem is:

$$\begin{aligned}
 \text{Min} \quad & \sum_{w \in W} \sum_{r \in E_w} P_{rw}(\pi, x) h_{rw} \\
 & \sum_{r \in E_w} h_{rw} = d_w, & \forall w \in W, \\
 & \sum_{w \in W} \sum_{r \in E_w \cap R_l} h_{rw} \leq B_l, & \forall l \in I, \forall i \in I, \\
 & h_{rw} \geq 0, & \forall w \in W, \forall r \in E_w.
 \end{aligned}$$

$P_{rw}(\pi, x)$ denotes the cost of a given route which, once the service instances departure times π and consequently variables x are fixed, can be obtained as follows :

$$P_{rw}(\pi, x) = \sum_{i, j \in C(r)} (\pi_j - \pi_i + Tx_{ijk(i, j, r)} - \varepsilon_{ik(i, j, r)} + \varepsilon_{jk(i, j, r)}) c_w$$

This problem can be solved efficiently by standard multicommodity flow codes (Ahuja, Magnanti and Orlin, 1993).

On the other hand, when a feasible flow h is given the subproblem is:

$$\begin{aligned}
 \text{Min} \quad & \sum_{w \in W} \sum_{r \in E_w} \sum_{i, j \in C(r)} (\pi_j - \pi_i + Tx_{ijk(i, j, r)} - \varepsilon_{ik(i, j, r)} + \varepsilon_{jk(i, j, r)}) c_w h_{rw} \\
 & \pi_j - \pi_i + Tx_{ijk(i, j, r)} \geq (\varepsilon_{ik(i, j, r)} - \varepsilon_{jk(i, j, r)}), & \forall r \in E_w, \forall i, j \in C(r), h_{rw} > 0 \\
 & \pi_j - \pi_i \geq T_s, & \forall i, j \in I_s, \\
 & 0 \leq \pi_i < T, & \forall i \in I, \\
 & x_{ijk(i, j, r)} \in \{0, 1\}, & \forall w \in W, \forall r \in E_w, \forall i, j \in C(r).
 \end{aligned}$$

The hard part of this problem resides in the presence 0-1 variables x , however it has an interesting structure which can be exploited in devising solution methods. In fact, if we fix variables x , the problem is a network *potential* problem, which can be efficiently solved by network flow algorithms. For more details see (Malucelli, 1996) where a class of problems of this type is studied. As the usual size of the model in real life applications is rather small, the mixed integer subproblem can be approached by standard branch and bound algorithm using the linear relaxation. Other solution approaches to problems of this type can be found in (Carraraesi and Malucelli, 1993).

This decomposability property suggests an iterative procedure which converges towards low cost solutions: starting from a set of feasible departure times, the flow problem is solved yielding a flow which is used in the objective function of the mixed linear integer problem. This last problem is solved obtaining new departure times, and the procedure is iterated. It is easy to verify that at each iteration the value of the solution does not increase, as the solution

obtained at the previous iteration is still feasible. The procedure halts when no improvements in the objective function are obtained.

6. Computational results

We present here some results from a very preliminary experimentation on eight problems based on three different railway networks derived from the Spanish (Problems 1.1, 1.2, 1.3, 2.1 and 2.2) and the Italian (Problems 3.1, 3.2 and 3.3) railway freight transportation systems. From these results we can draw some suggestions for further work. In particular a deeper investigation needs to be made for devising a relaxation scheme yielding better lower bounds; in fact the lower bounds obtained so far are very loose, and most likely this is one of the causes of the low quality of the feasible solutions yielded by the network models. Apparently the quadratic model seems to be much more promising, although, due to the lack of good bounds, we cannot make any strong statement about its performance.

In Problems 1.1, 1.2. and 1.3 there are six stations with capacities varying from 150 to 300 cars, five tracks between stations with a capacity of 4 trains per unit of time, and three commodities with demand 150, 160 and 140 cars. The delay costs are equal to 10 for all the commodities. The problems have a different number of active services. In Problem 1.1 there are only two services with frequencies 9 and 3 per week, respectively; Problem 1.2 has four services with frequencies of 3 and 4, and Problem 1.3 has 5 services with frequencies varying from 1 to 3. All the service instances have capacity of 50 cars. Table 6.1 reports the lower bound obtained solving the Lagrangean relaxation (lb), the value of the solution yielded by the flow model (fs) and by the quadratic model (qs).

	lb	fs	qs
Problem 1.1	0	2800	0
Problem 1.2	0	0	0
Problem 1.3	0	2800	0

Table 6.1

Problems 2.1 and 2.2 are slightly larger: nine stations (capacities from 250 to 400), ten arcs (capacity 6 trains), ten commodities (demand varying from 113 to 334, costs varying from 10 to 60). Problem 2.1 has 15 services with frequencies between 1 and 7, and Problem 2.2 has 19 services with frequencies between 1 and 6. All the services instances have capacity of 50 cars. Table 6.2 reports the results for these two problems.

	lb	fs	qs
Problem 2.1	38618	69445	57695
Problem 2.2	72596	103875	132675

Table 6.2

In the third group of examples, we considered a portion of the Italian railway freight transportation system: twelve stations (capacities ranging between 750 and 1250 cars), 16 tracks (capacity 12 trains per hour), and 20 services with frequencies between 1 and 7, and capacity between 20 and 30 cars. In all the problems we considered 10 commodities. Problems 3.1, 3.2 and 3.3 differ for the demand: in Problem 3.1 the demand ranges between 20 and 222 cars per commodity, in Problem 3.2 we increased the demand of 10% with respect to Problem 3.1, thus it ranges between 22 and 244, and in Problem 3.3 we increased the demand of 20% with respect to Problem 3.1, thus it ranges between 24 and 266. The costs vary between 10 and 30. The results are reported in Table 6.3.

	lb	fs	qs
Problem 3.1	0	12980	1550
Problem 3.2	0	22610	2010
Problem 3.3	1250	32090	5300

Table 6.3

Recall that all the models do not consider explicitly the track capacity. All the solutions produced by the heuristics have been tested, however none of them violates these constraints; hence they did not require to be adjusted.

The heuristic based on the quadratic model give the best results, except for Problem 2.2 where the flow algorithm have a better performance. Note that the gap between lower bound and the value of the heuristic solution is always quite large. It should be interesting to investigate whether this gap depends on the poor quality of the lower bound or on the quality of the heuristic solutions.

To get a better idea of the goodness of the solution found by our approaches, for Problems 3.1, 3.2 and 3.3, the ones derived from the italian railway network, we have compared the objective function values obtained by the algorithms with the values derived from the analysis of the current timetable. The values obtained for the current timetable are: 47200, 64880 and 92060, respectively. Thus the timetable obtained through our models appears to be definitely better than the current one.

We have also compared the additional cost due to the waiting times in stations with the *net travel cost*. The net travel cost is computed as the minimum theoretical travel time for each

commodity (i.e. relaxing the stations, tracks and services capacities, and not considering the connection time between services) multiplied by the number of cars and the cost coefficient related to the commodity. Tables 6.4, 6.5 and 6.6 report the percent value due to the delay in station cost with respect to the ideal travel time, for lb, fs and qs.

	lb	fs	qs
Problem 1.1	0%	18.7%	0%
Problem 1.2	0%	0%	0%
Problem 1.3	0%	18.7%	0%

Table 6.4

	lb	fs	qs
Problem 2.1	13.9%	25.0%	20.8%
Problem 2.2	26.2%	37.4%	47.8%

Table 6.5

	lb	fs	qs
Problem 3.1	0%	9.6%	1.2%
Problem 3.2	0%	15.3%	1.4%
Problem 3.3	0.8%	19.8%	3.3%

Table 6.6

The net travel cost is 15000 for problems 1.1, 1.2 and 1.3, and 277530 for problems 2.1 and 2.2; while it is 134860, 148170 and 161970 for problems 3.1, 3.2 and 3.3 respectively.

It should be noted that the percentage of the delay cost computed by qs for problems of larger size (3.1, 3.2 and 3.3) is very small, which provides further evidence of the goodness of the solution obtained from the quadratic model.

All the programs have been implemented in C++ and run on a workstation HP-9000-712. For the solution of the mixed linear integer problems and the multicommodity flow we used CPLEX 3.0. The optimization of the code was not the aim of this work, and, in fact, the computational times are quite high. The solution of the network flow model (both lower bound and heuristic) took from 30 minutes to 10/11 hours. However, more than 90% of this time was spent in solving multicommodity flow problems. The use of a more efficient code would drastically curtail the computational time. The heuristic for the quadratic model is more efficient: it took less than one second for the small examples, and at most 45 minutes for the largest one (Problem 3.3).

7. Conclusions

We have considered the problem of determining the services' schedules together with the goods' routes, in a railway freight transportation system, in order to minimize the delay costs due to poor synchronization.

This seems a problem only marginally dealt with in the literature, although quite relevant in practice. Here we have presented two alternative models leading to two solution algorithms.

Although preliminary and on a limited set of test problems, our experimentation suggests that the second approach is quite promising; in fact, in our problems it has yielded good solutions at rather low computational cost.

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