A non linear multicommodity network design approach to solve a location-allocation problem in freight transportation

A. Cavallet
F. Malucelli
R. Wolfler Calvo

Politecnico di Milano - Italy

Problem and data have been kindly provided by DANZAS

• activated international terminals

- - - international lines

—— internal collection/distribution

PROBLEM DEFINITION

The company is reorganizing the international transportation:

- **which terminals have to be closed and where to open new terminals**
- **size of the terminals**

- **assign international lines to terminals**
- **assign local customers to terminals**

- **evaluate the introduction of inter-terminal lines**

COST ANALYSIS

collection/distribution carried out by third parties

$\Rightarrow$ the costs are linear in the transported volume

international transportation carried out by DANZAS

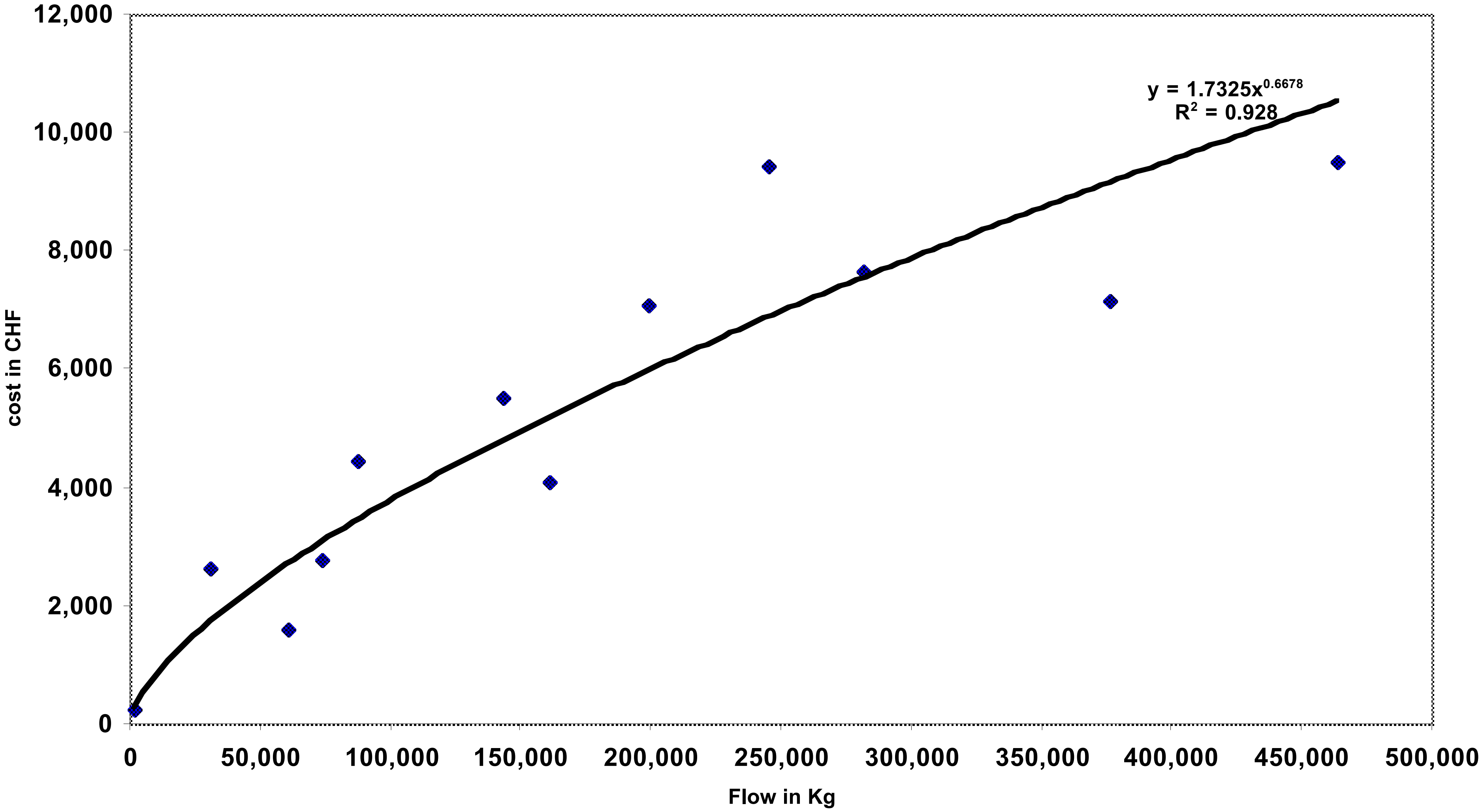
$\Rightarrow$ concave costs (economies of scale)

different shapes depending on the lengths

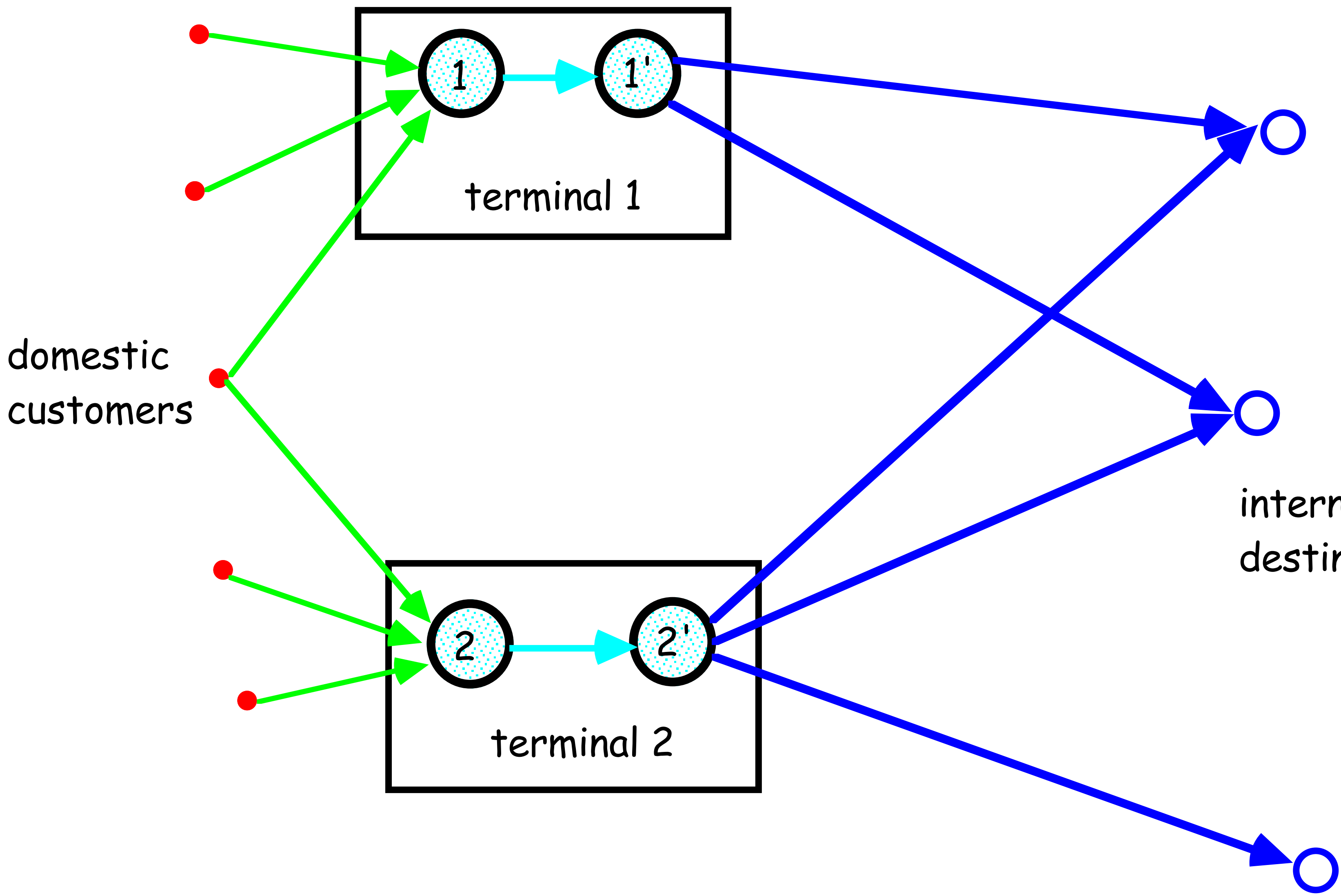
internal flow in terminals

$\Rightarrow$ concave costs

International transportation costs

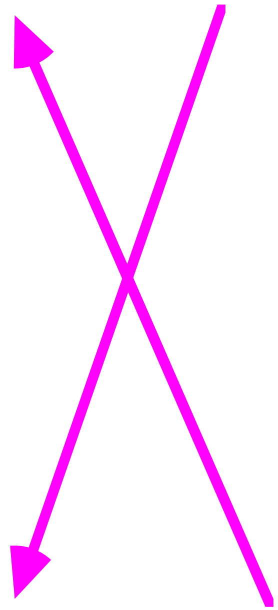


FLOW MODEL



1
1'
terminal 1
domestic customers
2
2'
terminal 2
international destinations

FLOW MODEL



inter-terminal
flow

FLOW MODEL

commodities jk (domestic origin j, international destination k)

d_{jk} = volume of goods to be transported from j to k

x^{jk}_{jh} = amount of flow of commodity jk going to terminal h

$x^{jk}_{hh'}$ = amount of flow of commodity jk inside terminal h

$x^{jk}_{h'k}$ = amount of flow of commodity jk going to destination k from terminal h

z_h = 1 if terminal h is activated, 0 otherwise

FLOW MODEL

$$\min \sum_{j}\sum_{h} f_c^{jh}\Big(\sum_{k} x_{jh}^{jk}\Big) + \sum_{h} f_m^{h}\Big(\sum_{jk} x_{hh'}^{jk}, z_h\Big) + \sum_{k}\sum_{h} f_i^{hk}\Big(\sum_{j} x_{h'k}^{jk}\Big)$$

$E^{jk}x^{jk} = \delta^{jk}$ for each commodity jk [flow conservation]

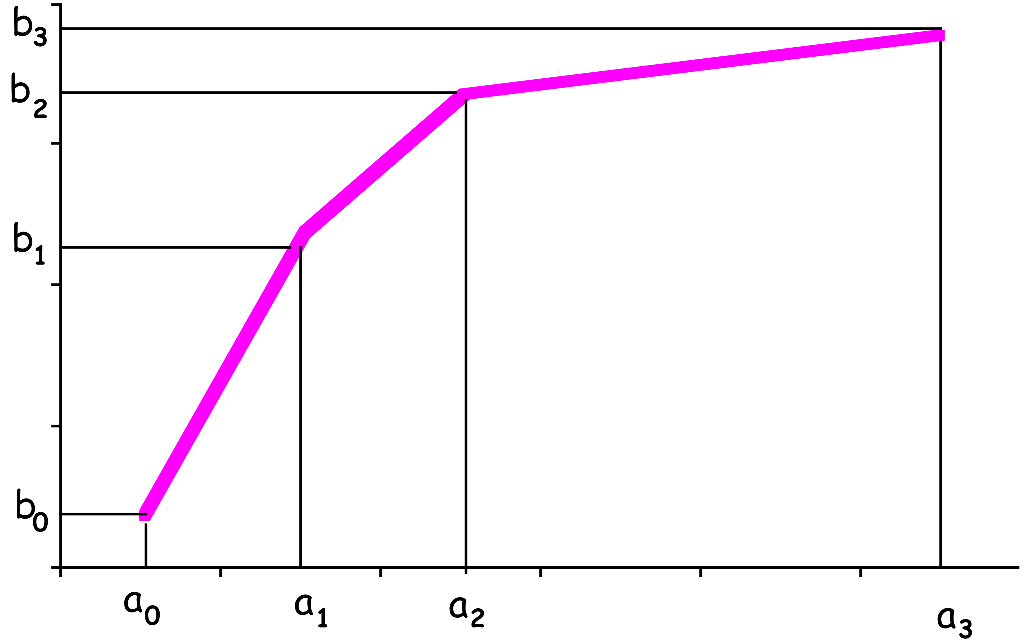
$x_{hh'}^{jk} \leq u_h z_h$ for each commodity jk and terminal h [design]

E^{jk} is the node/arc incidence matrix related to commodity jk

δ^{jk} is the demand vector

u_h is an upper bound of the flow passing by terminal h

LINEARIZATION OF SEPARABLE COST FUNCTIONS



$x = \sum_{l=0}^{k} a_l \xi_l$; $f(x) = \sum_{l=0}^{k} b_l \xi_l$; $\sum_{l=0}^{k} \xi_l = 1$; $\sum_{l=0}^{k-1} \eta_l = 1$;

$\sum_{j=0}^{l} \eta_j \geq \sum_{j=l+1}^{k} \xi_j \geq \sum_{j=l+1}^{k} \eta_j$ for $1 \leq l \leq k-2$

$0 \leq \xi_0 \leq \eta_0$; $0 \leq \xi_k \leq \eta_{k-1}$; $\eta_l \in \{0,1\}$ for $l=1,\ldots,k-1$

x is a convex combination of two consecutive interval endpoints

formulation "locally ideal" [Padberg]

"ASSIGNMENT" MODEL

In the previous model the flow of one commodity can split between two or more terminals (even though it is not convenient).

$$x_{jhk} = \begin{cases} 1 & \text{commodity } jk \text{ is assigned to terminal } h \\ 0 & \text{otherwise.} \end{cases}$$

the flow collected by h from customer j is given by:

$$\sum_{k} d_{jk} x_{jhk}$$

the flow inside terminal h is given by:

$$\sum_{jk} d_{jk} x_{jhk}$$

the flow on international line hk is given by

$$\sum_{j} d_{jk} x_{jhk}$$

"ASSIGNMENT" MODEL

$$\min \sum_{j} \sum_{h} f_c^{jh}\left(\sum_{k} d_{jk} x_{jhk}\right) + \sum_{h} f_m^{h}\left(\sum_{jk} d_{jk} x_{jhk}, z_h\right) + \sum_{k} \sum_{h} f_i^{hk}\left(\sum_{j} d_{jk} x_{jhk}\right)$$

$\sum_{h} x_{jhk} = 1$ for each commodity jk

$x_{jhk} \leq z_h$ for each commodity jk, for each terminal h

$x_{jhk}, z_h \in \{0,1\}$

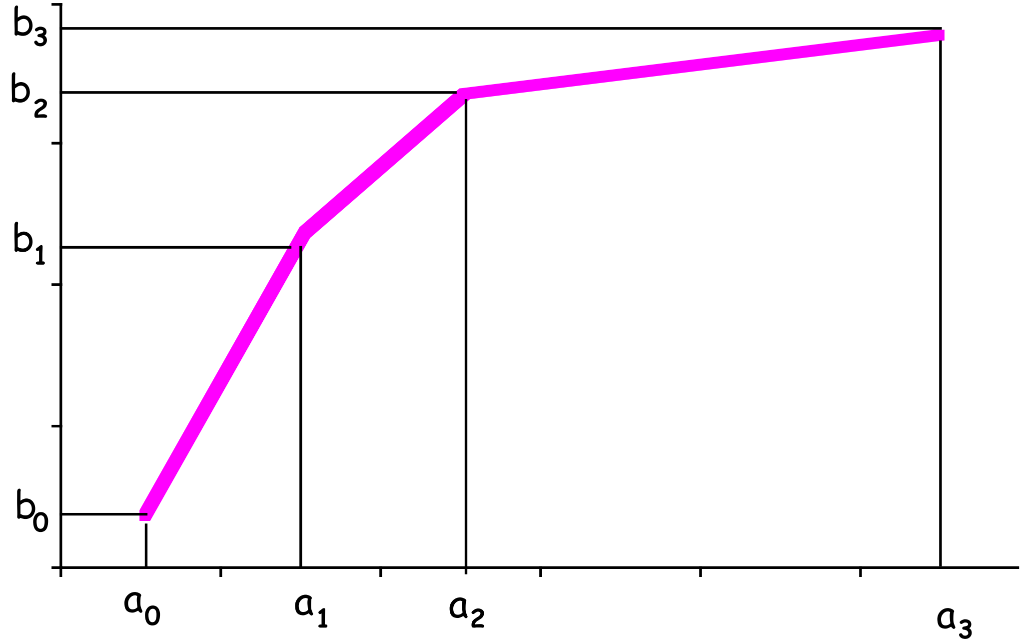
"ASSIGNMENT" MODEL

Inter-terminal flows

$$x_{jhrk} = \begin{cases} 1 & \text{commodity } jk \text{ uses terminals } h \text{ and } r \text{ in the order} \\ 0 & \text{otherwise.} \end{cases}$$

the constraints are modified accordingly

LINEARIZATION OF SEPARABLE COST FUNCTIONS (2)

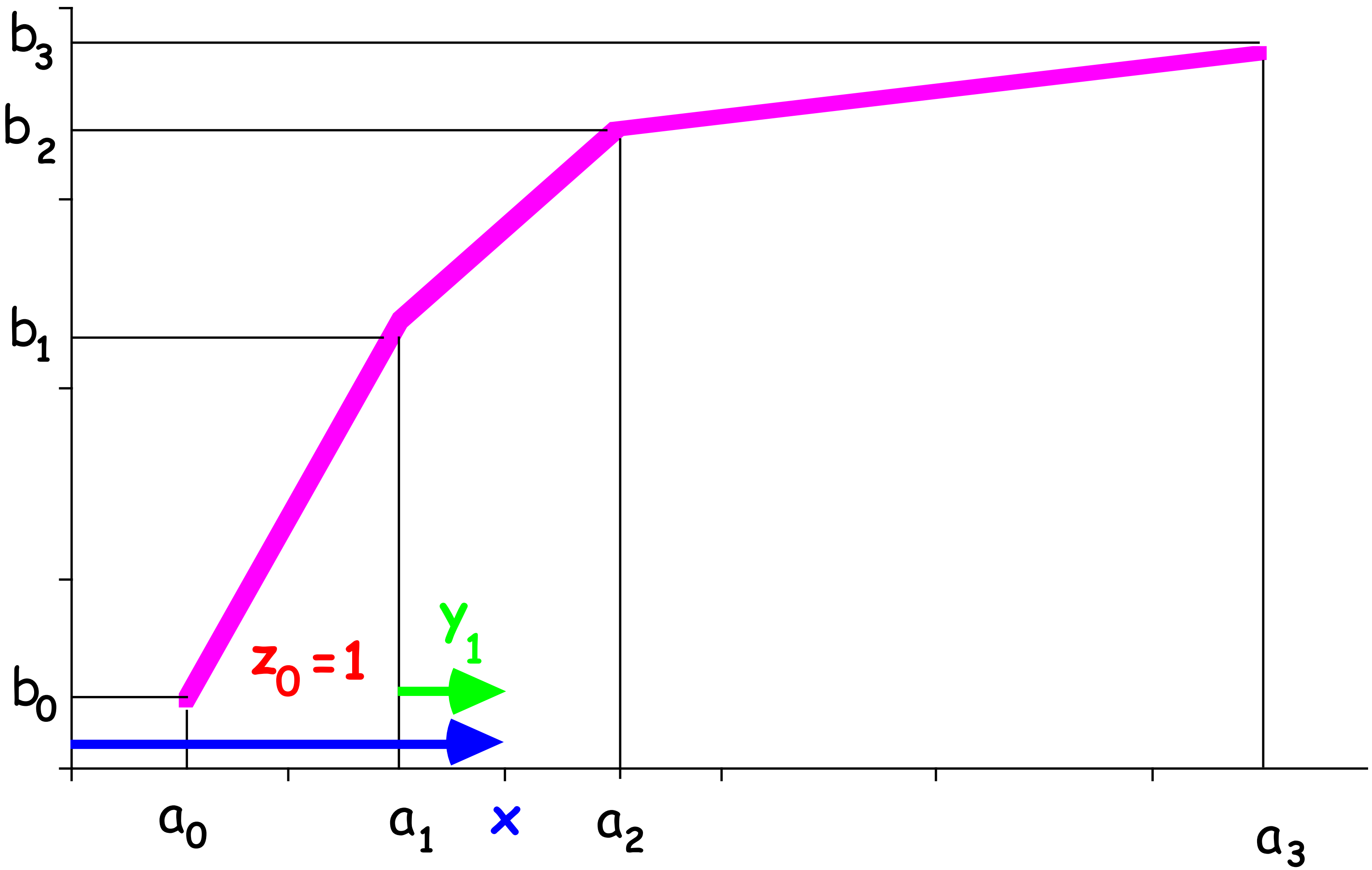


$x = a_0 + y_1 + \ldots + y_k$

$$f(x) = b_0 + \frac{b_1 - b_0}{a_1 - a_0} y_1 + \ldots + \frac{b_k - b_{k-1}}{a_k - a_{k-1}} y_k$$

$0 \le y_l \le a_l - a_{l-1}$

$y_l \ge (a_l - a_{l-1})\, z^l, \quad y_{l+1} \le (a_{l+1} - a_{l1})\, z^l, \quad$ for $l = 1, \ldots, k-1$



$$1 \geq z_1 \geq z_2 \geq \ldots \geq z_{k-1} \geq 0$$

Also this formulation is "locally ideal" [Padberg]

VALID INEQUALITIES

let S be a subset of commodities and h a terminal such that

$$\sum_{jk \in S} d_{jk} > a_l^h$$

if all the commodities in S are routed through terminal h

$$\Rightarrow z_l^h = 1$$

$$\sum_{jk \in S} x_{jhk} - |S| + 1 \leq z_l^h$$

separable by solving small knapsack problems

COMPUTATIONAL RESULTS

4 instances derived from real data provided by DANZAS

domestic areas / terminals / international destinations

	"assignment" model		"flow" model	
	# var.	# constr.	# var.	# constr.
3 / 2 / 2	37	110	26	21
6 / 4 / 5	206	241	624	540
15 / 10 / 10	1930	2214	2036	920
60 / 9 / 10	1930	2190	1084	734

COMPUTATIONAL RESULTS

	"assignment" model			"flow" model	
	LP	LP+VI	ILP	LP	ILP
3 / 2 / 2	177781	209239	209239	172265	209239
3 / 2 / 2 / IT	177781	209239	209239	172265	209239
6 / 4 / 5	145788	156479	158268	114608	158268
6 / 4 / 5 / IT	122955	137777	158268	114608	158268
15 / 10 / 10	5755093	5805112	6255961*	5487518	6249885*
15 / 10 / 10 / IT	5497962	5542283	6045015*	5243240	6081742*
60 / 9 / 10	1218631	1231530	1408750	1093086	1408750
60 / 9 / 10 / IT	1218631	1231485	1408750	1093086	1408750

ILP solved with CPLEX 6.6

*best integer after 1000 seconds (about 1-3% from optimum)

COMPUTATIONAL RESULTS

	"assignment" model		"flow" model
	LP	LP+VI	LP
3 / 2 / 2	15.0%	0.0%	17.7%
3 / 2 / 2 / IT	15.0%	0.0%	17.7%
6 / 4 / 5	7.9%	1.1%	27.6%
6 / 4 / 5 / IT	22.3%	12.9%	27.6%
15 / 10 / 10	8.0%	7.2%	12.2%
15 / 10 / 10 / IT	9.0%	8.3%	13.8%
60 / 9 / 10	13.5%	12.6%	22.4%
60 / 9 / 10 / IT	13.5%	12.6%	22.4%

Comparison with the best solution obtained with 9 open terminals

	optimal solution	9 terminals	gap
60 / 9 / 10	1408750	1572002	11.6%

COMPUTATIONAL TIMES

	"assignment" model			"flow" model	
	LP	LP+VI	ILP	LP	ILP
3 / 2 / 2	0.01	0.01	0.00	0.01	0.01
3 / 2 / 2 / IT	0.01	0.01	0.01	0.01	0.02
6 / 4 / 5	0.04	0.25	0.37	0.04	0.11
6 / 4 / 5 / IT	0.37	0.06	0.84	0.16	0.42
15 / 10 / 10	0.99	4.56	1000.00	0.36	1000.00
15 / 10 / 10 / IT	68.28	9.94	1000.00	8.05	1000.00
60 / 9 / 10	2.17	11.27	582.00	0.68	8.97
60 / 9 / 10 / IT	60.10	41.06	1968.00	16.12	424.00