# Flexible many-to-few + few-to-many <br> an almost personalized transit system 

T. G. Crainic UQAM and CRT Montréal
F. Errico - Politecnico di Milano
F. Malucelli - Politecnico di Milano
M. Nonato - Università di Ferrara
http://www.elet.polimi.it/people/malucell

## "Personalized" transit systems

Motivations

- Offer a competitive transportation w.r.t. the private one: capture additional demand
better serve population needs
cover larger areas
- Sustainability
reduce the operational costs
increase the resource utilization
- Integration with traditional transportation systems
from the users point of view
from the management point of view


## Dial a Ride systems

Users ask for personalized rides (door-to-door service) similar to a taxi service

They are served collectively similar to a bus service

Initially devised to meet needs of users with reduced mobility
Extended to deal with "low demand" areas or periods residential outskirts, night service ...

## Fixed Line

## DAR

known itinerary and timetable
no reservation is needed
one vehicle covers a small area
low service quality
no decision problems during service
network design phase
variable itinerary and timetable
accessed only through reservation
one vehicle covers a large area
good service quality
difficult decision problems for pick-up and delivery
no network design
no integration with the fixed lines
competition with taxi operators
localization devices are needed

## Demand Adaptive System <br> An attempt to conjugate Fixed Lines with DAR

- Lines with compulsory stops and possible deviations upon reques $\dagger$
- Flexibility in timetables
- Traditional users can still access the service in compulsory stops (passive users)
- Users that make reservations have a better level of service (active users)
- Vehicle and driver management can be integrated with traditional services


## Building block: 1 flexible line

compulsory stops

$$
\left[a_{1}, b_{1}\right]
$$

## $\left[a_{5}, b_{5}\right]$


$\left[a_{4}, b_{4}\right]$
time windows

The bus passes by the compulsory stops within the time windows

## Building block: 1 flexible line



The bus passes by an optional stop if a request of transportation is issued

## Single line - single tour case: off-line operation decision problem

## Given:

a line (compulsory stops, time windows, optional stops)
a set of requests $R$
travel costs and times, "benefits" of serving requests

Select a subset of requests and define the vehicle itinerary
so that
the time windows constraints are satisfied the difference between total benefits and costs is minimized

## Notation

request $r \in \boldsymbol{R}: r=(s(r), d(r))$ pair of boarding and alighting stops with benefit $u(r)$ :
segment $h=1, \ldots, n$ : subgraph between two consecutive compulsory stops $f_{h-1}$ and $f_{h}$
time windows [ah, $b_{h}$ ] for each compulsory stop $f_{h}$
path $p \in P_{h}$ : feasible path from $f_{h-1}$ to $f_{h}$ with cost $c(p)$ and travel time $\tau(p)$

Variables
$y_{r}$ : request selection variable
$z_{p}$ : path selection variable
$t_{h}$ : starting time from $f_{h}$

$$
\begin{aligned}
& \max \sum_{r \in R} u(r) y_{r}-\sum_{h=1}^{n} \sum_{p \in P_{h}} c(p) z_{p} \\
& y_{r} \leq \sum_{p \in P_{h}} \delta_{s}(r), p z_{p} \quad \forall r: s(r) \text { is in segment } h, h=1, \ldots, n \\
& y_{r} \leq \sum_{p \in P_{h}} \delta_{d}(r), p z_{p} \quad \forall r: d(r) \text { is in segment } h, h=1, \ldots, n \\
& \sum_{p \in P_{h}} z_{p}=1 \quad h=1, \ldots, n \\
& t_{h}+\sum_{p \in P_{h}} \tau(p) z_{p} \leq t_{h+1} \quad h=1, \ldots, n-1 \\
& t_{n}+\sum_{p \in P_{n}} \tau(p) z_{p} \leq b_{n+1} \\
& a_{h} \leq t_{h} \leq b_{h} \quad h=1, \ldots, n \\
& y_{r} \in\{0,1\} \\
& \forall r \in R \\
& z_{p} \in\{0,1\} \\
& \forall p \in P_{h}, h=1, \ldots, n
\end{aligned}
$$

## Solution approaches

## Upper bound

- Lagrangean decomposition of "coupling" constraints
- Lagrangean relaxation of "consecutive times" constraints


## Heuristic algorithms

- basic entities: paths
- pool of "promising" paths for each segment updated dynamically approximation of $P_{h}$
- multistart greedy randomized adaptive algorithms
- tabu search algorithms
- hybrid algorithms


## Excerpts of computational results

Winnipeg network
10 segments, 25 optional stops per segment
time windows between 60 to 120 seconds
250 requests
100 seconds runs

|  | upper bound | multistart | basic TS | hybrid |
| :--- | :--- | :--- | :--- | :--- |
| W1 | 279 | 278.61 | 277.55 | 278.61 |
| W3 | 211 | 207.72 | 208.83 | 208.70 |
| W5 | 227 | 217.45 | 213.12 | 219.39 |
| W7 | 228 | 214.03 | 218.16 | 216.19 |

Stockholm network: "easy instances"

## Convergence: W5


time

## Designing a flexible line

- Topological level
selection of compulsory stops
selection of optional stops
definition of segments
- Temporal level
time windows width
time difference between consecutive compulsory stops depending on the segment width (i.e., maximum deviation from the direct path)

Different criteria for the urban or extra-urban setting

Example: urban area in Ferrara (Italy)


Line structure


## Urban line: design parameters

10 segments, 57 optional stops
decided in collaboration with the transportation company

Total travel time: 1 h
imposed by the company

Time window width: from 2 min to 8 min

## Urban line: some results

| distribution | served req | $Q$ | average LOS |
| :--- | :---: | :---: | :---: |
| $0 \%$ comp. | $87 \%$ | 0.97 | 1.48 |
| $30 \%$ comp. | $89 \%$ | 0.97 | 1.49 |
| $50 \%$ comp. | $90 \%$ | 0.98 | 1.50 |

average results on 5 instances of 20 requests
Q: solution profit/upper bound LOS: actual travel time / "ideal travel time"
"ideal travel time": minimum travel time from the origin to the destination of the request passing by the compulsory stops satisfying the time windows

Increasing instance sizes


## Extra-urban case

Compulsory stops in the center of villages

The optional stops are not uniformly distributed (concentrated around the villages)


Question: how to partition the optional stops into segments?

1) Partition the optional stops as in the urban setting


Difficult if there is not a unique way to get in and to get out of the village
2) Duplicate the optional stops of the village:
drop-off stops belong to the incoming segment pick-up stops belong to the outgoing segment


A bus can pass by a compulsory village stop twice (first: drop-off, second pick-up)
$s(r)$ and $d(r)$ may belong to the same segment, though time windows constraints make the paths passing by $d(r)$ before $s(r)$ infeasible
3) Duplicate the compulsory stops of the village center

A "village segment" goes from the first copy of the compulsory stop to the second copy and includes all optional stops of the village


A bus passes by a village center compulsory stop twice the first time it drops-off passengers (both "passive" and "active") the second time it picks-up "passive" passengers

Example: the area "Basso ferrarese" around Copparo


## Extra-urban line: design parameters

Optional stops partition method no. 3 in 3 villages
11 segments, 174 optional stops corresponding to the existing bus stops

Total travel time: 1 h 30 min (very short!) imposed by the company

Time window width: from 2 min to 10 min

## Extra-Urban line: some results

| distribution | served req | Q | average LOS |
| :--- | :---: | :---: | :---: |
| $0 \%$ comp. | $68 \%$ | 0.92 | 1.92 |
| $30 \%$ comp. | $84 \%$ | 0.94 | 1.95 |
| $50 \%$ comp. | $86 \%$ | 0.95 | 1.99 |

average results on 5 instances of 20 requests

Q: solution profit/upper bound LOS: actual travel time / "ideal travel time"
"ideal travel time": minimum travel time from the origin to the destination of the request passing by the compulsory stops satisfying the time windows

## Extra-urban case: modified network



## Extra-Urban line: served requests

| distribution | original line | modified line |
| :--- | :---: | :---: |
| $\mathbf{0 \%}$ comp. | $68 \%$ | $90 \%$ |
| $30 \%$ comp. | $84 \%$ | $91 \%$ |
| $50 \%$ comp. | $86 \%$ | $92 \%$ |

Increasing instance sizes


## Multiple lines - multiple tours

 Example

## Example: integrated system



- Optional stops
- Compulsory stops


## Assumptions:

- fixed synchronization at compulsory stops
- negotiation for possible displacement in time or space
- taxi rides

The route of a passenger is described by a sequence of vehicle legs
optional - compulsory - ... - compulsory - optional
only the first and the last legs must pass by optional stops
the definition of the intermediate legs is not important for the passenger as the synchronization is fixed

## Mathematical model

The passenger itinerary can be summarized by the pair of terminal legs that compose it
$W_{r}=$ set of pairs of boarding and alighting legs corresponding to feasible routes for request $r$
$u_{w}$ : benefit related with pair $w$ of request $r$
$\sigma$ : index of segment
route selection variables:
$x_{w}= \begin{cases}1 & \text { if request } r \text { is routed through pair } w \text { in } W_{r} \\ 0 & \text { otherwise } .\end{cases}$
$y_{r}= \begin{cases}1 & \text { if a taxi ride is used for request } r \\ 0 & \text { otherwise. }\end{cases}$
$\max \sum_{r \in R} \sum_{w \in W_{r}} u_{w} x_{w}-\sum_{\sigma \text { segment }} \sum_{p \in P_{\sigma}} c(p) z_{p}-\sum_{r \in R}$ taxi $y_{r}$

$$
\begin{array}{ll}
x_{w} \leq \sum_{\sigma \text { segment }} \sum_{p \in P_{\sigma}} \delta_{s(r), p} z_{p} & \forall r \in R, \forall W_{r} \\
x_{w} \leq \sum_{\sigma \text { segment }} \sum_{p \in P_{\sigma}} \delta_{d(r), p} z_{p} & \forall r \in R, \forall W_{r} \\
\sum_{w \in W_{r}} x_{w}+y_{r}=1 & \forall r \in R
\end{array}
$$

$$
\sum_{p \in P_{\sigma}} z_{p}=1
$$

for each segment $\sigma$
$t_{\sigma}+\sum_{p \in P_{\sigma}} \tau(p) z_{p} \leq t_{\sigma^{\prime}} \quad$ for each consecutive segments $\sigma$ and $\sigma^{\prime}$
$a_{\sigma} \leq t_{\sigma} \leq b_{\sigma} \quad$ for each segment $\sigma$
$x_{w}, y_{r}, z_{p} \in\{0,1\}$

## Solution approaches

Lagrangean decomposition of coupling constraints ( $\lambda$ )
Lagrangean relaxation of service constraints ( $\mu$ )

Evaluation of the Lagrangean function $\Phi(\lambda, \mu)$ :
solution of many single line problems for each line occurrence (almost all requests involve only one optional stop)

Parallel approaches

A feasible solution can be generated easily

## Conclusions

The proposed system provides a good flexibility maintaining the features of a traditional fixed line system: traditional users and users who ask explicitly for a ride may share the system

Limited technological requirements
Low costs

Integration with traditional transportation systems

Efficient algorithms supporting the managing decisions

