Planning maximum capacity Wireless Local Area Networks

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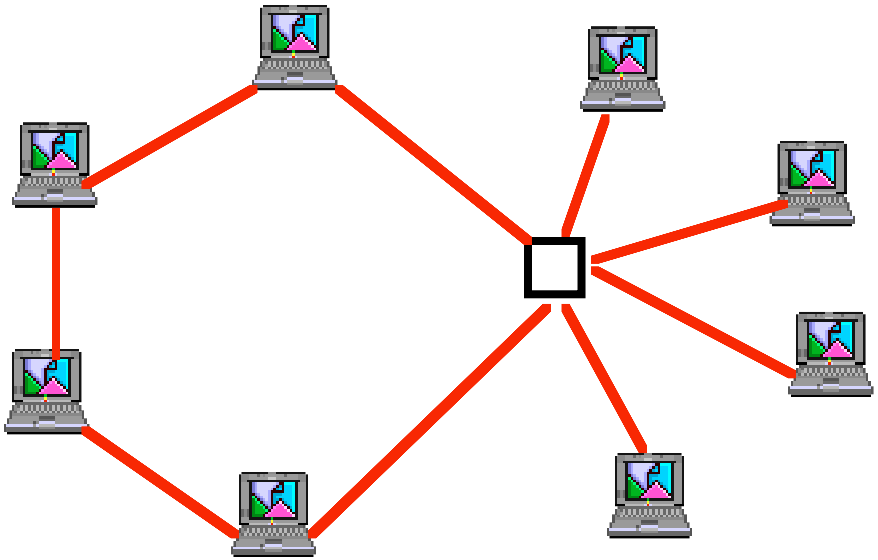
Outline

- Application
- 3 combinatorial optimization problems
- Complexity issues
- Hyperbolic formulations and solution approaches
- Quadratic formulations and solution approaches
- Linearization and model strenghthening
- Preliminary computational results
- Concluding remarks

Wireless Local Area Networks (WLANs)

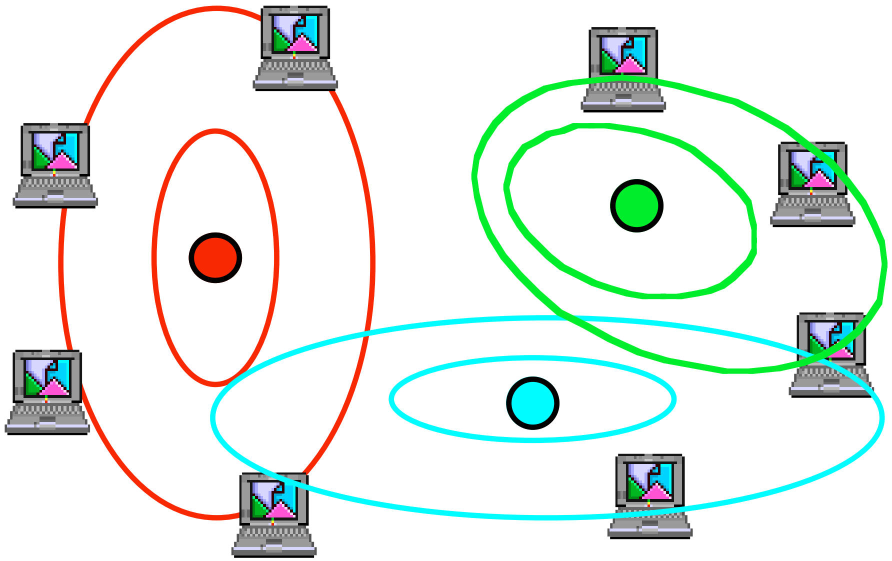
(Cabled) Local Area Networks



- Dramatic size increase
- Difficult cable management
- Cannot cope with users' mobility

$\Rightarrow$ Introduction of wireless connections

Wireless Local Area Networks (WLANs)



Users connected to the network via antennas (access points, hot spots)

WLANs allow: to substitute cables in offices and departments (easier and more flexible management)

to provide network services in public areas (airports, business districts, hospitals, etc.)

at very low cost

WLAN planning

$J = \{1,\ldots,n\}$ candidate sites where antennas can be installed

$I = \{1,\ldots,m\}$ "test points" (TPs) or possible users positions

For each $j \in J$:

$I_j \subseteq I$ subset of test points covered by antenna j

Goal: select a subset of candidate sites $S \subseteq J$

with covering constraints:

each test point must be covered by at least one antenna

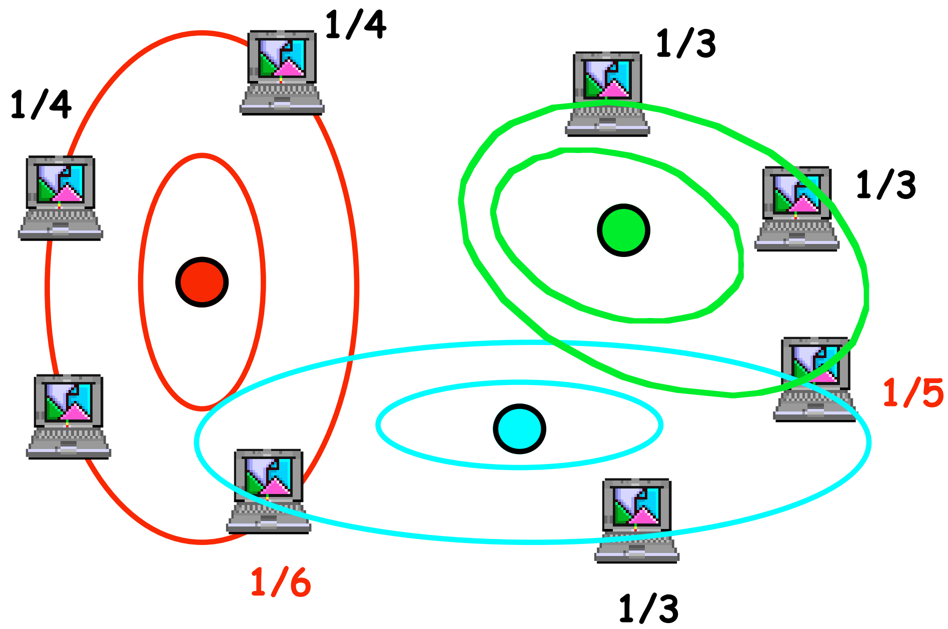
without covering constraints:

a test point is not necessarily covered

Solution quality measures

Transmission protocol: a user can "talk" if all interfering users are "silent"

"Talking probability" = 1/(# of the interfering users)



Network capacity = sum of the "talking probabilities" of all users

Objective functions

For any $S \subseteq J$, let $I(S)$ denote the subset of users covered by S

- Network capacity

$$c(S) = \sum_{i \in I(S)} \frac{1}{|\cup_{j \in S : i \in I_j} I_j|}$$

- Network fairness

$$f(S) = \min_{i \in I} \frac{1}{|\cup_{j \in S : i \in I_j} I_j|}$$

Intuitively solutions with small non-overlapping subsets should be privileged

Combinatorial Optimization problems

Maximum capacity unconstrained WLAN

$$P: \{\max c(S): S \subseteq J\}$$

Maximum capacity covering WLAN

$$PC: \{\max c(S): S \subseteq J, \cup_{j \in S} I_j = I\}$$

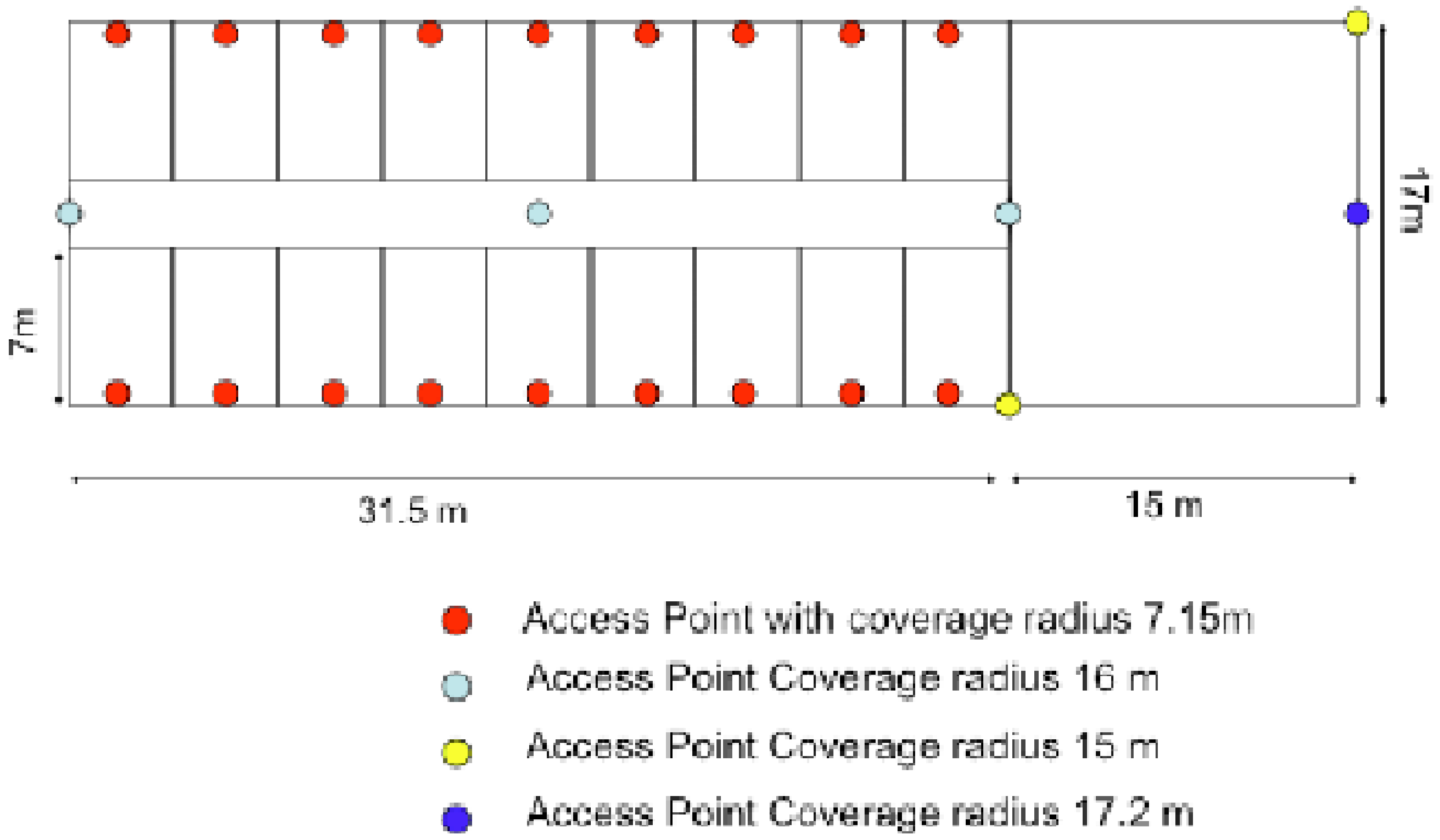
Maximum fairness WLAN

$$PF: \{\max f(S): S \subseteq J\}$$

PF implies full coverage, since any solution covering all users dominates those not covering some users (which have fairness =0)

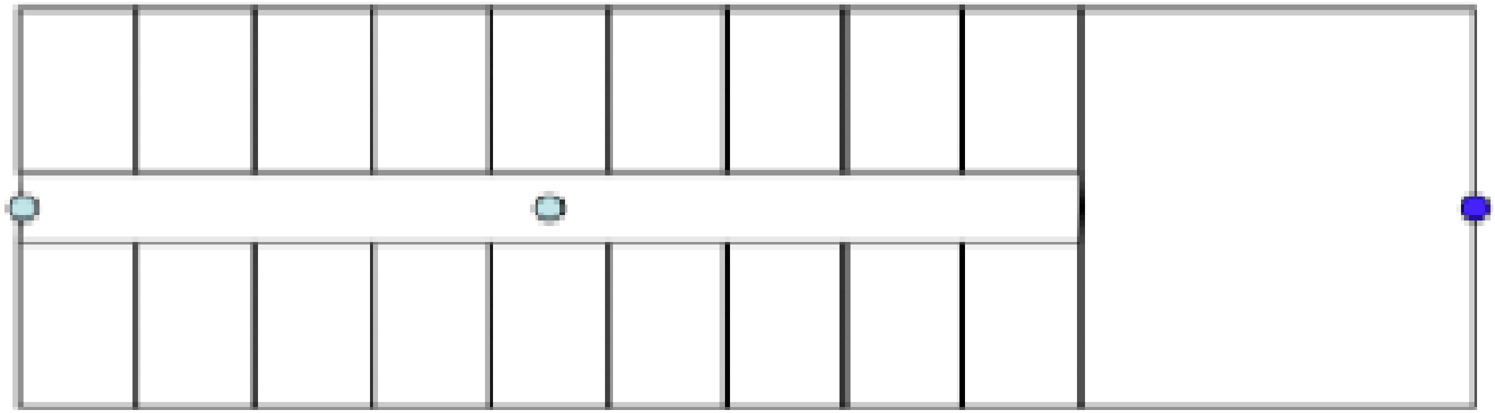
Example: third floor of our department

Candidate sites

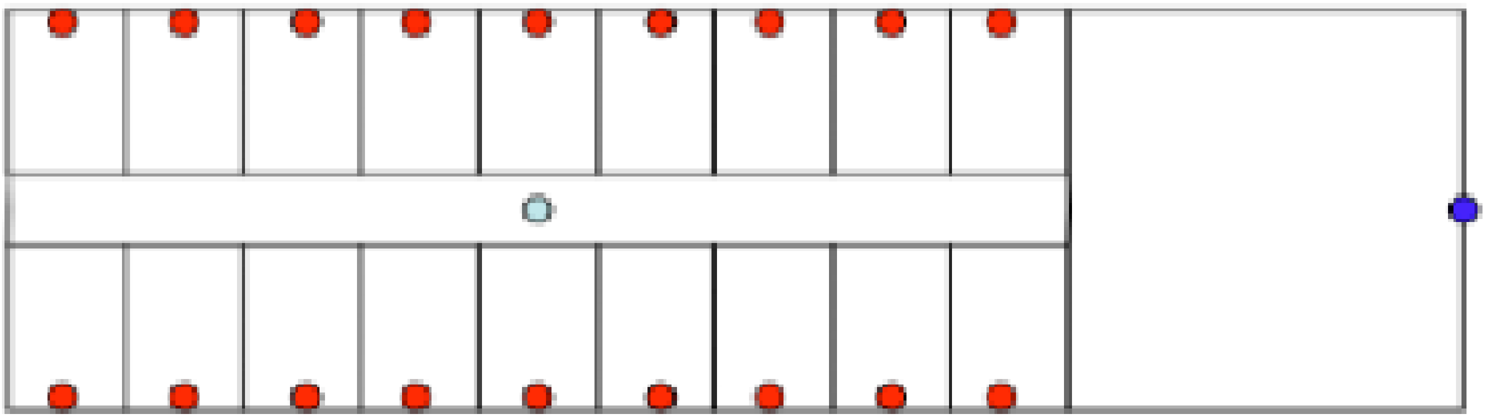


Test points uniformly distributed

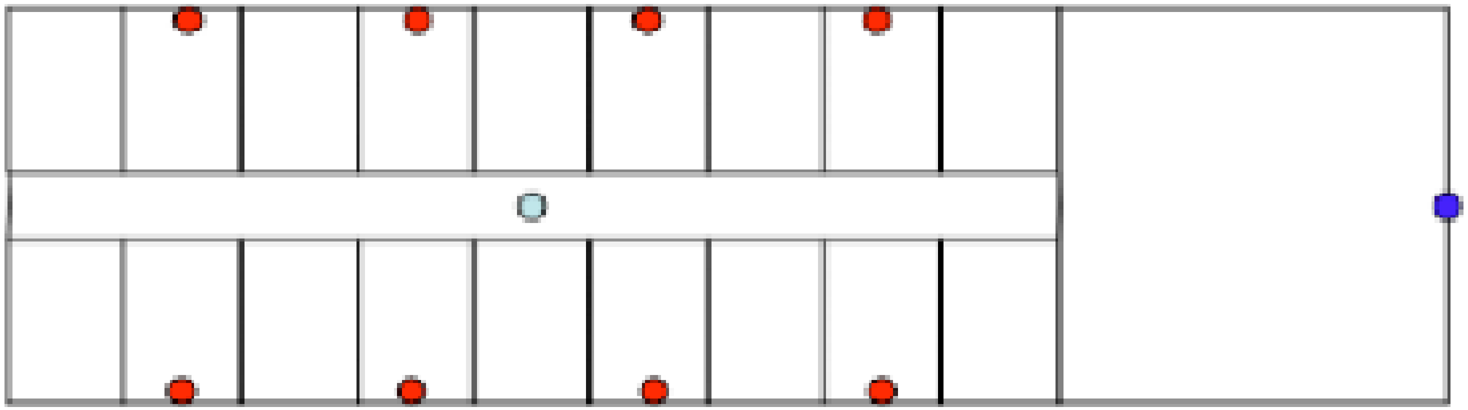
Minimum cardinality set covering



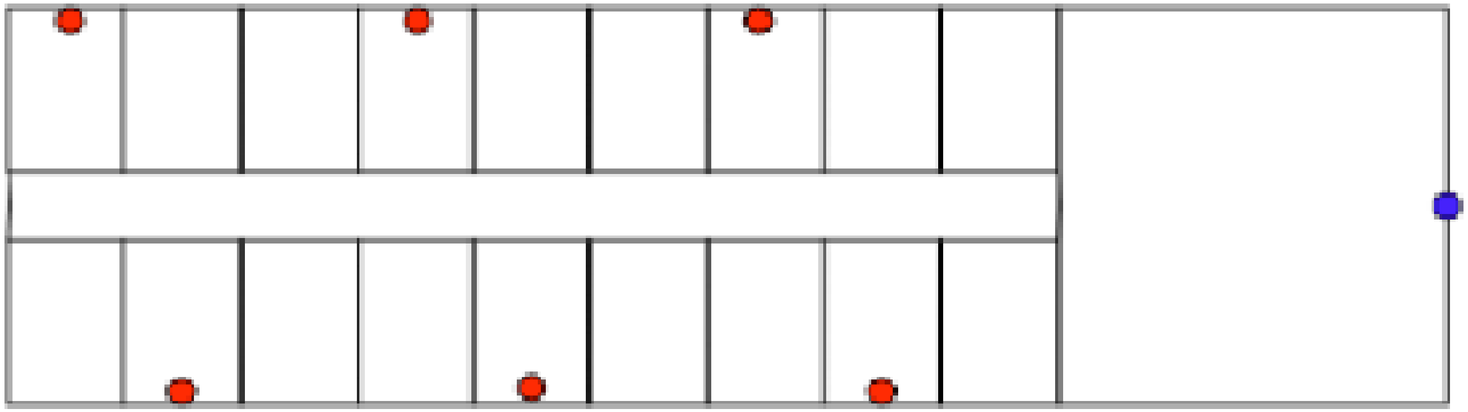
Practitioner solution (dense)



Practitioner solution (sparse)



Maximum capacity solution (PC)



Numerical results

	# Access Points	Capacity	Efficiency
Min. card. Set Covering	3	1.913	0.638
Practitioner dense	20	2.448	0.122
Practitioner sparse	10	2.582	0.258
Maximum capacity	7	5.649	0.807

Efficiency = Capacity/(# Access Points)

Computational complexity

Proposition: *P*, *PC*, and *PF* are NP-hard

Reduction

Exact Cover by 3-sets:

Given a set X ($|X|=3q$) and a collection $\mathcal{C}$ of n 3-element subsets C_j, $j=1,\dots,n$, of X, does $\mathcal{C}$ contain an exact cover of X, i.e., $\mathcal{C}' \subseteq \mathcal{C}$ s.t. every element of X occurs in exactly one element of $\mathcal{C}$?

$$I = X, \quad J = \{1,\dots,n\}, \quad \{I_1,\dots,I_n\} = \mathcal{C}, \qquad S\ \{\ j:\ C_j \in \mathcal{C}'\ \}$$

P (PC) has a solution S with $c(S)=q$ iff $\mathcal{C}'$ is an exact cover

PF has a solution S with $f(S)=1$ iff $\mathcal{C}'$ is an exact cover

Mathematical Programming Formulations

Data: users/subsets incidence matrix

$$a_{ij} = \begin{cases} 1 & \text{if } i \in I_j,\ j \in J \\ 0 & \text{otherwise} \end{cases}$$

Variables:

$$x_j = \begin{cases} 1 & \text{if } j \in S, \\ 0 & \text{otherwise} \end{cases}$$ selection of subset I_j

$$y_{ih} = \begin{cases} 1 & \text{if } i \text{ and } h \text{ appear together in a selected subset,} \\ 0 & \text{otherwise} \end{cases}$$ union definition

$$z_i = \begin{cases} 1 & \text{if } i \text{ is covered,} \\ 0 & \text{otherwise} \end{cases}$$ coverage of user i

Max capacity covering WLAN (PC)

$$PCH: \max \sum_{i \in I} \frac{1}{\sum_{h \in I} y_{ih}}$$

$$\sum_{j \in J} a_{ij} x_j \geq 1 \quad \forall\, i \in I \qquad \text{full coverage}$$

$$a_{ij}\, a_{hj}\, x_j \leq y_{ih} \quad \forall\, i,h \in I,\ \forall\, j \in J \qquad \text{definition of } y_{ih}$$

$$y_{ih} \geq 0 \quad \forall\, i,h \in I$$

$$x_j \in \{0,1\} \quad \forall\, j \in J$$

Hyperbolic sum 0-1 constrained problem

Max capacity unconstrained WLAN (P)

PH: max $\displaystyle\sum_{i\in I} \frac{z_i}{\sum_{h\in I} y_{ih}}$

$\displaystyle\sum_{j\in J} a_{ij}\, x_j \geq z_i \quad \forall\, i \in I$ — definition of z_i

$a_{ij}\, a_{hj}\, x_j \leq y_{ih} \quad \forall\, i,h \in I,\ \forall\, j \in J$ — definition of y_{ih}

$0 \leq z_i \leq 1 \quad \forall\, i \in I$

$y_{ih} \geq 0 \quad \forall\, i,h \in I$

$x_j \in \{0,1\} \quad \forall\, j \in J$

Hyperbolic sum 0-1 constrained problem

Max fairness WLAN (*PF*)

PFH: max $\min_{i \in I} \dfrac{1}{\sum_{h \in I} y_{ih}}$

$$\sum_{j \in J} a_{ij}\, x_j \geq 1 \qquad \forall\, i \in I \qquad \text{full coverage}$$

$$a_{ij}\, a_{hj}\, x_j \leq y_{ih} \qquad \forall\, i,h \in I, \forall\, j \in J \qquad \text{definition of } y_{ih}$$

$$y_{ih} \geq 0 \qquad \forall\, i,h \in I$$

$$x_j \in \{0,1\} \qquad \forall\, j \in J$$

Hyperbolic bottleneck 0-1 constrained problem

Solving Hyperbolic formulations

Problems *PH* and *PCH* cannot be solved by standard techniques

nor the algorithms studied for Hyperbolic unconstrained 0-1 problems [Hansen, Poggi de Aragão, Ribeiro 90; 91] can be extended to the constrained case

$$\max \{\sum_i \frac{a_{io} + \sum_j a_{ij}x_j}{b_{io} + \sum_j b_{ij}x_j}, \; x_j \in \{0,1\}\}$$

Problem *PFH* can be solved by a sequence of mixed integer linear systems

PFH: $\max \{ \beta : \beta \in SFH(\beta) \}$ Fairness $\beta \in [0,1]$

SFH(β):

$$1 \geq \beta \left(\sum_{h \in I} y_{ih} \right) \quad \forall\ i \in I$$

$$\sum_{j \in J} a_{ij} x_j \geq 1 \quad \forall\ i \in I$$

$$a_{ij}\, a_{hj}\, x_j \leq y_{ih} \quad \forall\ i,h \in I$$

$$y_{ih} \geq 0 \quad \forall\ i,h \in I,\ x_j \in \{0,1\} \quad \forall\ j \in J$$

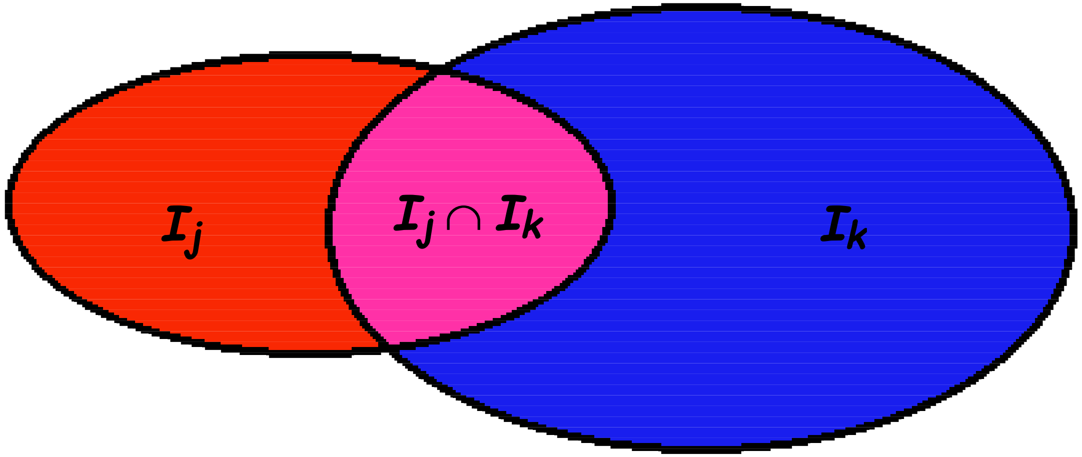
Optimal β can be found by binary search (solving a sequence of *SFH*(β))

Otherwise let $\alpha = 1/\beta$ and minimize α

Quadratic formulation (1)

$$c_j = \sum_{i \in I_j} \frac{1}{|I_j|} = 1$$

$$q_{jk} = \frac{|I_j \cap I_k|}{|I_j \cup I_k|} - \frac{|I_j \cap I_k|}{|I_j|} - \frac{|I_j \cap I_k|}{|I_k|} \qquad (-1 \le q_{ij} \le 0)$$



Quadratic formulation (1)

$$QPC: \quad \max \quad \frac{1}{2}xQx + cx$$
$$Ax \geq 1$$
$$x \in \{0,1\}^n$$

$$QP: \quad \max \quad \frac{1}{2}xQx + cx$$
$$x \in \{0,1\}^n$$

Linear contribution: capacity of a non overlapping subset

Quadratic contribution: penalty due to the overlapping of two subsets

Quadratic formulation (1)

QPC and *QP* are equivalent to *PC* and *P* if each element belongs to at most 2 subsets

In the other cases *QPC* and *QP* underestimate network capacity

QP can be approached by pseudoboolean techniques

QPC is a Quadratic Set Covering problem

- Semidefinite Programming
- Combinatorial optimization approaches
- Bounding techniques derived from QAP (e.g. Gilmore and Lawler)

Quadratic formulation (1)

P_j: subproblem obtained by fixing x_j=1

$$w_j = \max \frac{1}{2} \sum_{k \in J} q_{jk} x_k$$

$$\sum_{k \in J} a_{ik} \geq 1 \qquad \forall i \in I \setminus I_j$$

$$x_k \in \{0,1\} \qquad \forall\ k \in J$$

due to nonpositiveness of coefficients q_{jk} P_j is a Set Covering

$$W = \max \sum_{j \in J} (w_j + c_j)\ x_j$$

$$\sum_{j \in J} a_{ij} \geq 1 \qquad \forall i \in I$$

$$x_j \in \{0,1\} \qquad \forall\ j \in J$$

After some fixing, W can be computed by a Set Covering

Quadratic formulation (1)

Claim: *W* is an upper bound for *QPC*

It is an upper bound also when we use relaxations instead of computing the exact solution of the set covering problems

Quadratic formulation (2)

Tradeoff between network capacity and cost

$$p_{jk} = \frac{|I_j \,\Delta\, I_k|}{|I_j \cup I_k|} \qquad (0 \leq p_{jk} \leq 1)$$

approximate measure of the capacity: tends to favor non overlapping subsets

g_j = installation cost

$$QPC': \quad \max\{\tfrac{1}{2}xPx - \alpha\, gx : Ax \geq 1,\ x \in \{0,1\}^n\}$$

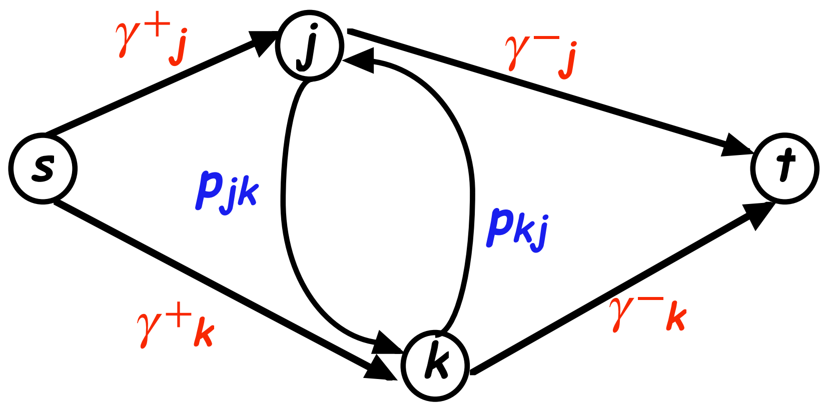
$$QP': \quad \max\{\tfrac{1}{2}xPx - \alpha\, gx,\ x \in \{0,1\}^n\}$$

tradeoff parameter $\alpha > 0$

Quadratic formulation (2)

QP' can be solved in polynomial time (min cut computation)

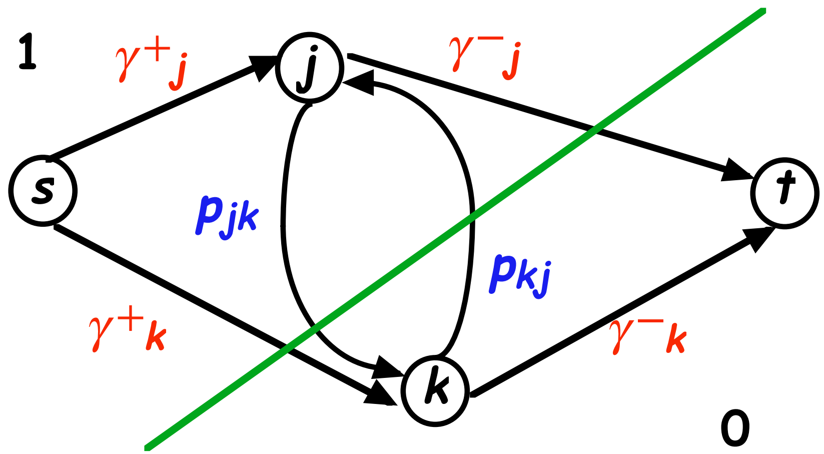
Auxiliary graph $G = (N, A)$ with capacities



$$\gamma_j^+ = \max\left\{0, \frac{1}{2}\sum_{k \in J} p_{jk} - \alpha\, g_j\right\}$$

$$\gamma_j^- = \max\left\{0, -\frac{1}{2}\sum_{k \in J} p_{jk} + \alpha\, g_j\right\}$$

The minimum capacity *s-t* cut corresponds to the solution x maximizing the objective function of QP' [Hammer 65]



Quadratic formulation (2)

The Lagrangian relaxation of *QPC'* can be solved efficiently

Minimization of a piecewise convex function

At each iteration the computation of a min cut gives the value of the Lagrangian function

Computational results Quadratic vs. Hyperbolic

Small instances ($|J| = 10$, $|I| = 100, 300$)

Subsets = circles in the plane (radii 50m, 100m, 200m)

Comparison of the objective functions:

Hyperbolic, Quadratic, Fairness, # installed access points

Exact solutions computed by enumeration

Simple heuristic algorithms

Average on 10 instances

Full coverage: exact solutions

cs=10	Fairness exact				Hyperbolic exact				Quadratic exact			
	#AP	Hyperbolic	Quadratic	fairness	#AP	Hyperbolic	Quadratic	fairness	# AP	Hyperbolic	Quadratic	fairness
R=50m	9.9			0.0509	9.9	9.3340	9.3229	0.0509	9.9	9.3340	9.3229	0.0509
R=100m	9.4			0.0358	9.4	7.4243	7.2618	0.0358	9.4	7.4243	7.2618	0.0358
R=200m	7.1			0.0193	7.0	4.3440	4.0155	0.0192	7.0	4.3381	4.0605	0.0191
R=50m	10.0			0.0179	10.0	9.3134	9.3054	0.0179	10.0	9.3134	9.3054	0.0179
R=100m	9.6			0.0122	9.6	7.4979	7.3711	0.0122	9.6	7.4979	7.3711	0.0122
R=200m	7.5			0.0063	7.5	4.2445	3.6443	0.0063	7.5	4.2146	3.6465	0.0063

Without covering constraints

cs=10	Hyperbolic exact			Quadratic exact		
	# AP	Hyperbolic	Quadratic	# AP	Hyperbolic	Quadratic
R=50m	9.8	9.3675	9.3675	9.8	9.3675	9.3675
R=100m	8.8	7.5682	7.4691	8.5	7.5465	7.5465
R=200m	5.9	4.7685	4.7685	5.9	4.7685	4.7685
R=50m	9.9	9.3318	9.3317	9.9	9.3318	9.3318
R=100m	8.9	7.6375	7.5433	8.5	7.6154	7.6108
R=200m	6.0	4.6865	4.6508	5.9	4.6778	4.6778

Full coverage: comparison exact and heuristic solutions

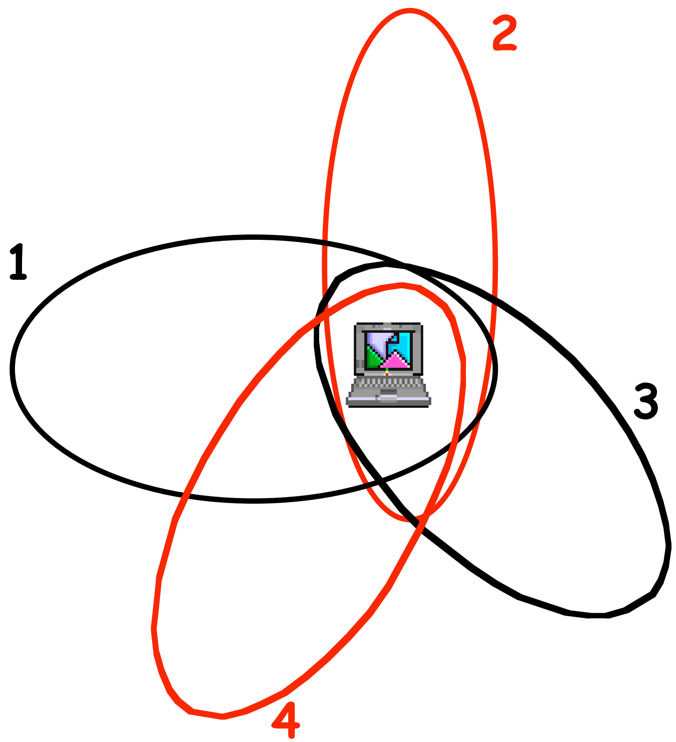
cs=10	Hyperbolic		Quadratic	
	exact	heuristic	exact	heuristic
R=50m	9.3340	9.3340	9.3229	9.3229
R=100m	7.4243	7.4243	7.2618	7.2618
R=200m	4.3440	4.3440	4.0605	4.0605
R=50m	9.3134	9.3134	9.3054	9.3054
R=100m	7.4979	7.4979	7.3711	7.3711
R=200m	4.2445	4.2445	3.6465	3.6375

Full coverage: comparison exact and heuristic solutions

cs=10	Hyperbolic		Quadratic	
	exact	heuristic	exact	heuristic
R=50m	9.3675	9.3675	9.3675	9.3675
R=100m	7.5682	7.5682	7.5465	7.5465
R=200m	4.7685	4.7685	4.7685	4.7685
R=50m	9.3318	9.3318	9.3318	9.3318
R=100m	7.6375	7.6375	7.6108	7.6108
R=200m	4.6865	4.6865	4.6778	4.6778

Linearization: idea

A test point i may be covered by different activated antennas



Introduce a 0-1 variable ξ_{ir} for each test point i and each subset r of possible activated antennas covering i

Exponentially many variables, depending on the cardinality of overlaps

Linearization: notation

J_i subset of candidate sites covering i

$S(i) = 2^{J_i} \setminus \{\emptyset\}$ set of antennas configurations covering i

we include the emptyset if the total coverage is not required

$J(r)$ subset of candidate sites of configuration r

For each configuration r in $S(i)$ we can compute the contribution of test point i to the total network capacity

$$K_{ir} = \frac{1}{\left|\cup_{j \in J(r)} I_j\right|}$$

(K_{ir} can be computed also according to the quadratic formulation)

Linearization: model

$$\max \sum_{i \in I} \sum_{r \in S(i)} K_{ir} \xi_{ir}$$

$$\sum_{r \in S(i)} \xi_{ir} = 1 \quad \forall\, i \in I$$ (one configuration per test point)

$$\sum_{r: j \in J(r)} \xi_{ir} = x_j \quad \forall\, i \in I,\ \forall\, j \in J_i$$

(consistency in configuration selection)

$$x_j \in \{0,1\} \quad \forall\, j \in J$$

$$\xi_{ir} \geq 0 \quad \forall\, i \in I,\ r \in S(i)$$

note that only x variables are binary

Instance generator

Generated on a geometric base (2D)

Avoided test points covered by a single candidate site

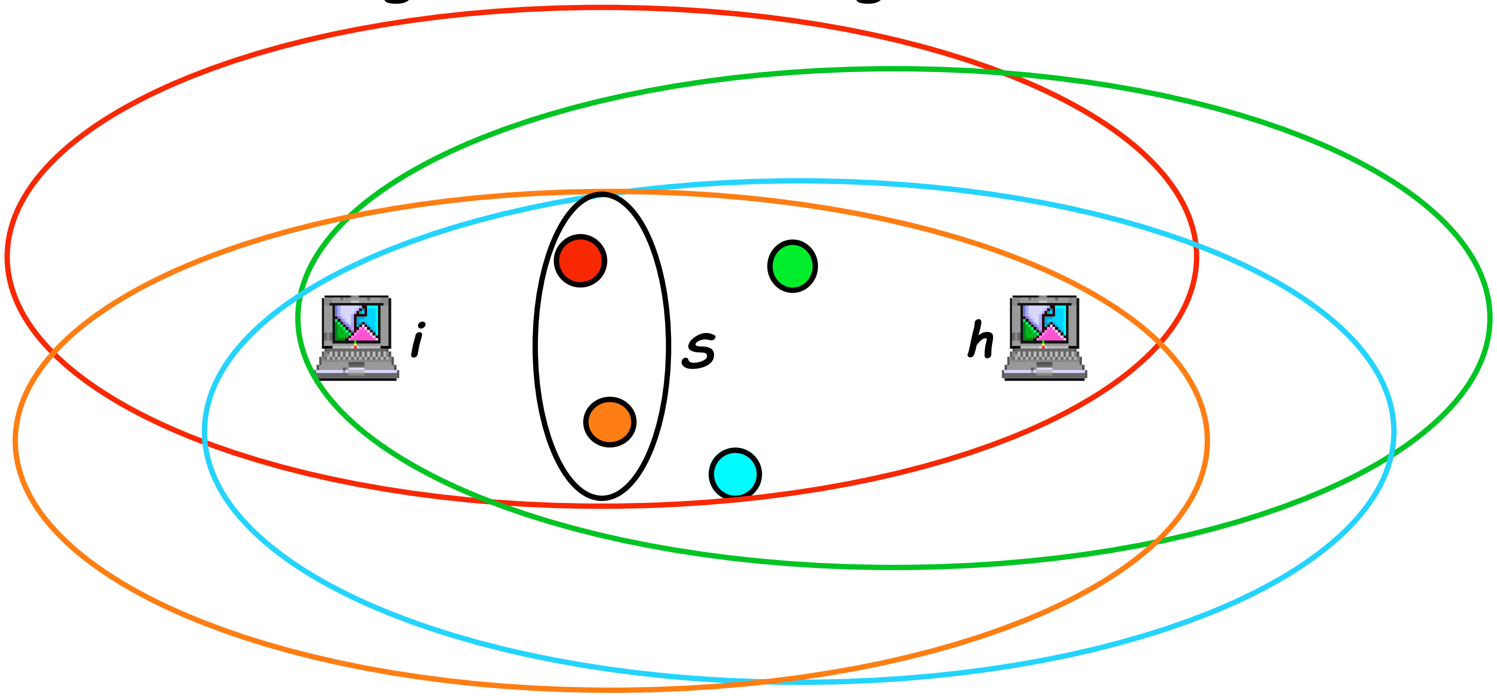
Computational results (1)

instance	QUADRATIC				HYPERBOLIC			
|J|/|I|/id	Integer		LP		Integer		LP	
	time	value	time	value	time	value	time	value
50/25/1	0.1	12.7667	0.0	13.1833	0.2	12.7667	0.0	13.3083
50/50/1	0.4	13.2075	0.0	13.2427	0.7	13.2075	0.0	13.7656
50/100/1	9.9	13.1375	0.5	13.9411	22.1	13.6384	0.6	14.5831
75/37/1	5.8	13.4762	0.2	14.4708	17.3	13.4762	0.2	14.8961
75/75/1	0.8	18.6944	0.0	19.2456	2.3	19.0724	0.1	20.3144
75/150/1	5.3	19.5738	0.1	20.0742	19.3	19.6275	0.2	20.6583
100/50/1	10.8	20.5762	0.5	21.2996	53.0	20.6556	0.3	21.8009
100/100/1	69.2	21.2211	3.4	21.9479	1138.7	21.4125	4.6	23.3422

times in seconds on a 2.8 GHx Xeon

Strenghthening equalities

Consider a pair of test points *i* and *h* and a subset *S* of candidates sites among those covering both *i* and *h*



The selected configurations in *S* for *i* and *h* must coincide

$$\sum_{r:\, j \in J(r) \cap S} \xi_{ir} = \sum_{r:\, h \in J(r) \cap S} \xi_{hr}$$

Computational results (2)

instance	QUADRATIC				HYPERBOLIC			
|J|/|I|/id	Integer		LP		Integer		LP	
	time	value	time	value	time	value	time	value
100/50/1	10.8	20.5762	0.5	21.2996	53.0	20.6556	0.3	21.8009
|S|=2	1.4	20.5762	0.8	20.5762	8.3	20.6556	1.3	20.7803
|S|=3	2.8	20.5762	1.0	20.5762	5.1	20.6556	2.2	20.6556
|S|=2/var 3	0.1	20.5762	0.1	20.5762	0.2	20.6556	0.1	20.6833
100/100/1	69.2	21.2211	3.4	21.9479	1138.7	21.4125	4.6	23.3422
|S|=2	22.0	21.2211	10.6	21.2211	993.9	21.4125	131.0	21.8300
|S|=3	40.9	21.2211	17.1	21.2211	584.3	21.4125	302.8	21.6012
|S|=2/var 3	0.7	21.2211	0.7	21.2211	4.1	21.3257	1.9	21.5599

var3: generated variables for subsets of at most 3 candidate sites

Concluding remarks

New interesting combinatorial optimization problems

Hyperbolic 0-1 formulations

Quadratic formulations (good approximation)

Linearization and strenghthening

Column generation?

Frequency assignment