Discrete models and algorithms for packet scheduling in smart antennas

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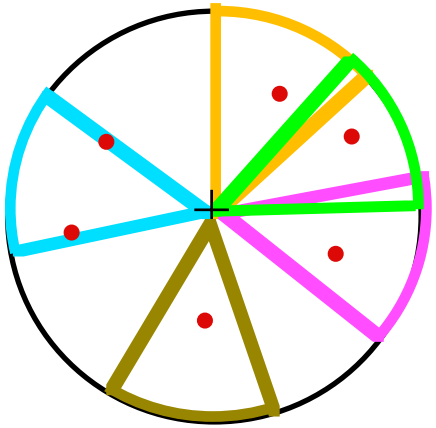
Smart Antennas

A smart antenna (adaptive antennas array) can be considered as a set of co-located directive antennas oriented via software (DSP, beam forming)



- beams of constant width (e.g. 12°)
- low interference (negligible among non intersecting beams)
- frequency reuse (Space Division Multiple Access scheme)
- possible combination with a Code Division Multiple Access (more users per beam)

Packet scheduling problems

Consider a smart antenna and users spatially distributed in its cell

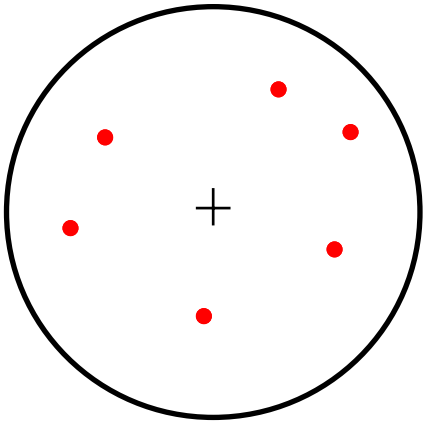
Each user must send/receive a number of packets

Combinatorial optimization problems:

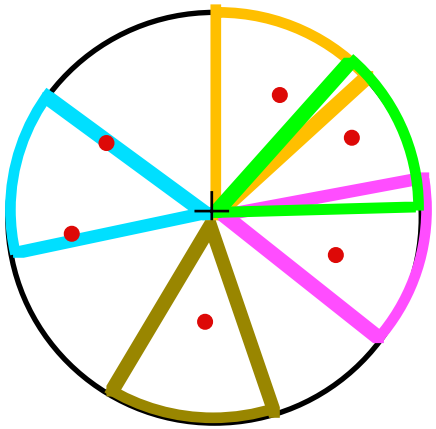
- Select the maximum number of non interfering users to be served in one time slot

- Partition the users in subsets of non interfering transmissions in order to minimize the number of slots

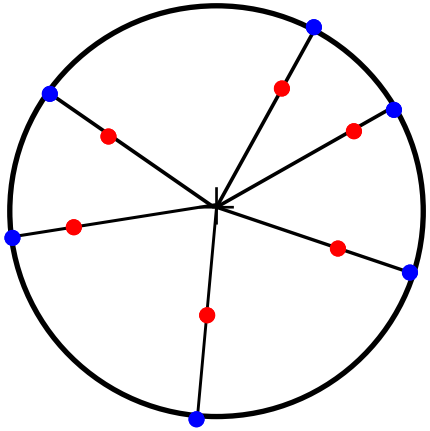
Circular arc model



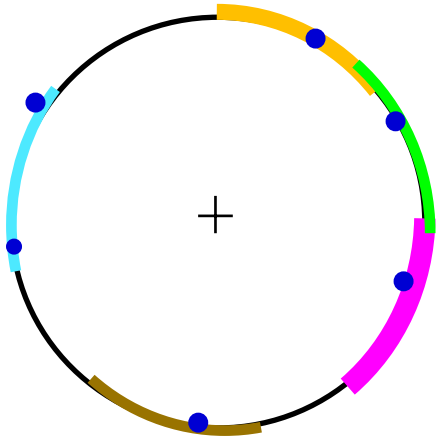
users in the cell



"placement" of beams and assignment of users



projection on the unit circumference



beams ≡ arcs

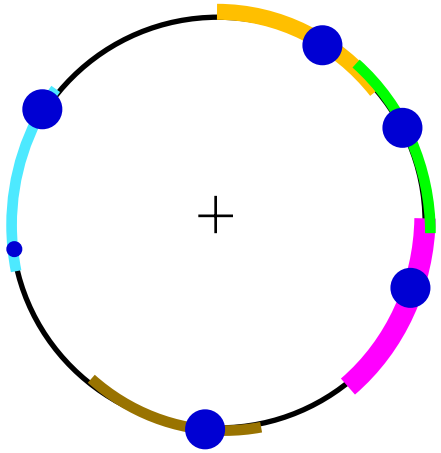
Data and notation

- $I = \{1, \ldots, n\}$ a set of points on the unit circumference
- ρ_i = distance from an arbitrary 0
- w_i = weight (priority) >0
- α = arc (beam) width
- distance between two points

$$||(i,j)|| = \min\{|\rho_i - \rho_j|, |1 - \rho_i + \rho_j|\}$$

α-arc-independent set of points

Subset of points $\bar{I}$, a set of arcs $A = \{A_i\}_{i \in \bar{I}}$ and a one-to-one assignment of points to arcs such that each point is cointained in exactly one arc



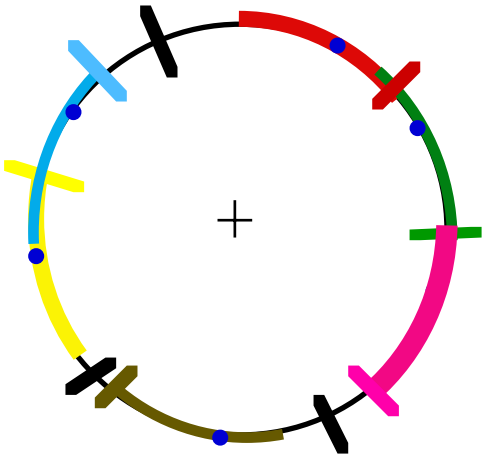
Arcs may overlap but each point must be contained in at most one arc

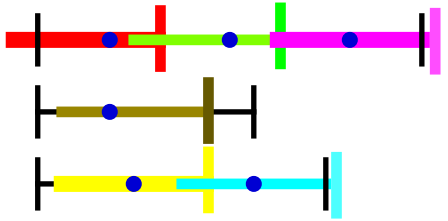
Infinite number of feasible solutions

Assumptions

- The distance between consecutive points in I is $< \alpha$

Otherwise the problem can be reduced to a problem on the line and/or decomposed into independent subproblems





- The circular-arc intervals are opened on the left

Problems

1) Max (weighted) α-arc-independent set

Given a set of points $I=\{1,\ldots,n\}$ on C and a real $\alpha > 0$, determine an α-arc-independent subset $\bar{I}$

Objectives:

a) maximize $|\bar{I}|$ (max cardinality)

b) maximize $\sum_{i \in \bar{I}} w_i$ (max priority)

2) Min partition into α-arc-independent sets

Given a set of points $I=\{1,\ldots,n\}$ on C and a real $\alpha > 0$, partition I into α-arc-independent subsets $M_1, M_2, \ldots, M_p \subseteq I$

Objective:

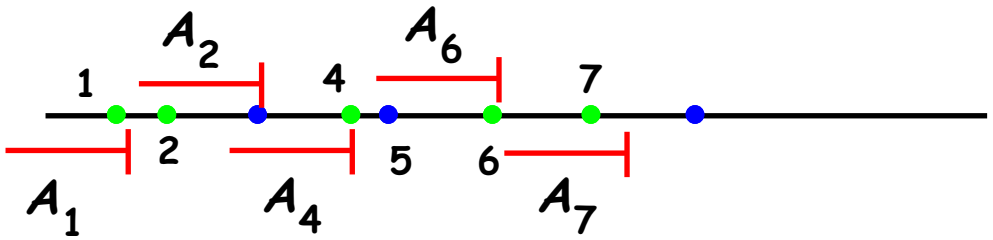
minimize p

Extremal solutions

Property: Given any α-*arc*-independent subset $\bar{I}$ with arc placement A, we can always consider the equivalent arc placement A' with all arcs shifted counterclockwise

Consider any two consecutive points i and j in $\bar{I}$, arc A_j has right endpoint in ρ_j or left (open) endpoint in ρ_i

Example of extremal solution



$\bar{I}=\{1,2,4,6,7\}$

The overall extremal solutions are finite
(but exponentially many)

A polynomial algorithm for the max weight α-arc independent subset

Based on the longest path computation on graph G

Node set: each node corresponds to a point selection

$N = N_1 \cup N_2$

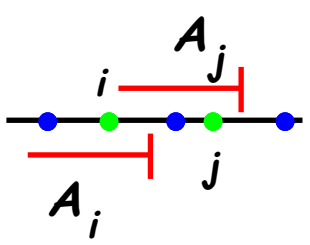
$N_1 = \{i: i \in I\}$

selection of point i and A_i with right endpoint in ρ_i

weight: w_i

$N_2 = \{(i,j): i,j \in I \text{ and } ||(i,j)|| < \alpha\}$

selection of point j and A_j with left endpoint in ρ_i



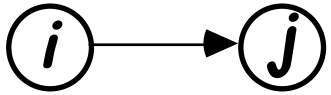
weight: w_j

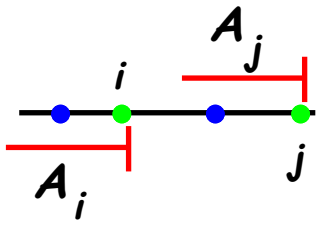
The node set can be partitioned into *Layers*:

$L_i = \{ i \} \cup \{(j,i) \in N_2\}$

any node in Layer L_i corresponds to the selection of point i and a suitable extremal placement of circular arc A_i

Arc set: compatibility between pairs of selections

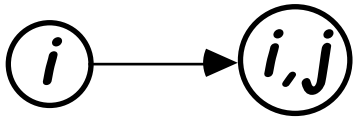


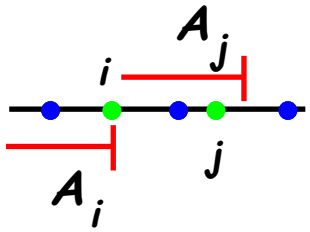


$\forall i,j \in I$:

$||(i,j)|| \geq \alpha$

$O(n^2)$

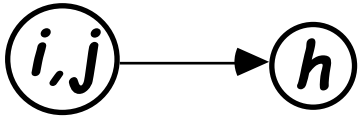


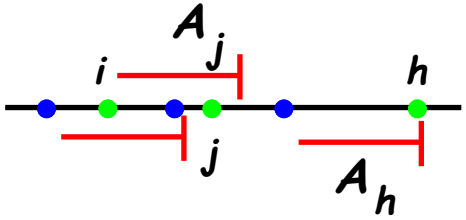


$\forall i,j \in I$:

$||(i,j)|| < \alpha$

$O(n^2)$

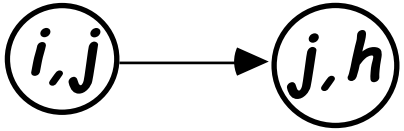


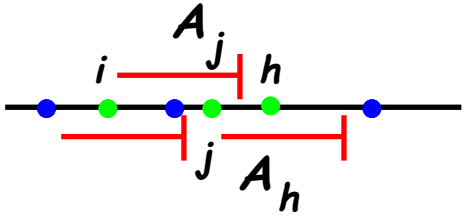


$\forall (i,j) \in N_2,\ h \in I$:

$||(h,j)|| \geq \alpha$

$O(n^2)$





$\forall (i,j) \in N_2,\ h \in I$:

$||(h,j)|| < \alpha$

$O(n^2)$

Any cycle P corresponds to an α-arc independent subset

- $\bar{I} = \{i: i \in P \cap N_1,$ or $(h,i) \in P \cap N_2\}$
- the placement of each circular arc is determined by the nodes in P
- each point in $\bar{I}$ is contained in exactly one circular arc

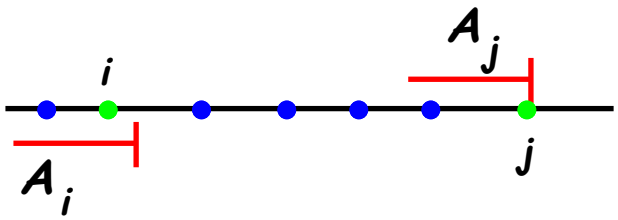
The "heaviest" cycle corresponds to one optimal solution

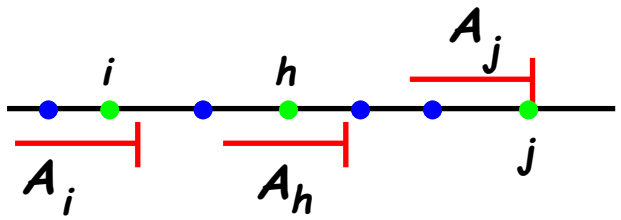
Dominance relation

Since $w_i > 0$ and the distance between consecutive points is $< \alpha$ the arc set can be reduced

All arcs corresponding to the selection of two points i and j $||(i,j)|| \geq 2\alpha$ can be dropped

The corresponding solution is dominated by any solution containing a point between i and j





How to compute the maximum weight cycle

Due to the dominance rule the number or graph arcs arriving in a layer or bypassing it is polynomially bounded

Idea of the algorithm

- Consider two consecutive points i and $i+1$ in I
  (those maximizing $||(i, i+1)||$)
- "Open" the graph between the corresponding layers
- Compute the heaviest paths from nodes of L_{i+1} and those of L_i (this can be done in polynomial time since the graph now is acyclic)
- Check if the cycles can be closed, in case eliminating the last or the first node
- Select the heaviest cycle

Computational results

n. of users	basic algorithm	improved algorithm
100	0.02	0
150	0.20	0
200	0.46	0
250	2.35	0.19
300	13.72	0.21
350	24.19	0.36
400	41.21	0.66
450	90.41	1.89
500	156.36	3.19

Computational times in seconds on a 1.2 GHz PC

Minimum partition into α-arc independent subsets

Consider graph G without reducing the arc set according to dominance rule

Finding the minimum partition is equivalent to finding the minimum cycle cover having a node in each subset L_i i=1,…,n

It can be formulated as a particular flow problem with side constraints

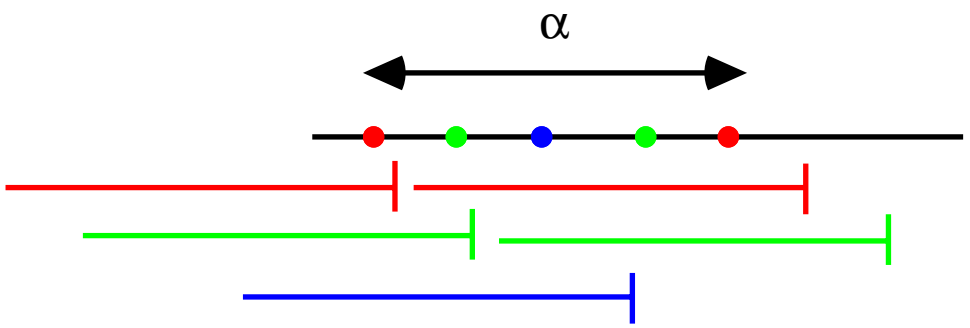
Is it polynomially solvable?

Minimum partition into α-arc independent subsets

Lower bound

Consider the α-wide portion of *C* containing the maximum number of points (*M*).

The partition of these points requires at least $\lceil M/2 \rceil$ α-arc-independent subsets



Greedy heuristic

Select each time the maximum cardinality α-arc-independent set of points

If the number of partitions is greater than the lower bound apply a local search phase

Computational results

n. of users	lower bound	greedy	+local search
80	4	4	-
110	4	5	4
140	7	7	-
170	6	6	-
200	8	8	-
215	7	8	7
250	10	10	-
300	10	10	-
400	13	14	13